Forcing axioms and model companionship results for set theory

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We present recent results on the model companions of set theory, placing them in the context of the current debate in the philosophy of mathematics. We start by describing the dependence of the notion of model companionship on the signature, and then we analyze this dependence in the specific case of set theory. We argue that the most natural model companions of set theory describe (as the signature in which we axiomatize set theory varies) theories of H_{κ^+}, as κ ranges among the infinite cardinals. We also single out $2^{\aleph_0} = \aleph_2$ as the unique solution of the Continuum problem which can (and does) belong to some model companion of set theory (enriched with large cardinal axioms).