Ramsey-theoretic statements over a weaker base theory

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The common base theory of reverse mathematics is the theory RCA_0 , which guarantees the existence of Δ_1^0 -definable sets and where mathematical induction for Σ_1^0 -formulae holds. In 1986, Simpson and Smith introduced a different base theory, RCA_0^* , where induction is weakened to Δ_1^0 -formulae. In more recent years Kołodziejczyk, Kowalik, Wong, Yokoyama started wondering about the strength of Ramsey's theorem over RCA_0^* . In this talk we concentrate on four well known consequences of Ramsey's theorem for pairs, namely the Ascending Descending Sequence principle ADS, the Chain/Antichain principle CAC, the Cohesive Set principle COH, the Cohesive Ramsey theorem for pairs CRT_2^2 . We measure the relative strength of these statements in three ways: (1) implications or non-implications among them over RCA_0^* (and over RCA_0^* plus negated Σ_1^0 -induction), (2) conservativity over RCA_0^* and (3) provable closure properties of the intersection of all Σ_1^0 -cuts.