Tractable depth-bounded approximations to First-Degree Entailment

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Many useful propositional logics are likely to be intractable, so we cannot expect a real agent to be always able to recognize in practice that a certain conclusion follows from a given set of assumptions. The "depth-bounded" approach to Classical Propositional Logic [6, 7, 8] provides an account of how this logic can be approximated in practice by realistic agents in two moves: i) providing a semantic and proof-theoretic characterization of a tractable 0-depth approximation, and ii) defining an infinite hierarchy of tractable k-depth approximations, which can be naturally related to a hierarchy of realistic resource-bounded agents, and admits of an elegant proof-theoretic characterization.

The logic of *First-Degree Entailment* (**FDE**) [1] admits of an intuitive semantics based on informational values [9, 4], which was put forward as the logic in which "a computer *should* think". These values are interpreted as four possible ways in which an atom p can belong to the present state of information of a computer's database, which in turn is fed by a set of equally "reliable" sources: **t** means that the computer is told that p is true by some source, without being told that p is false by any source; **f** means the computer is told that p is false but never told that p is true; **b** means that the computer is told that p is true by some source and that p is false by some other source (or by the same source in different times); **n** means that the computer is told nothing about the value of p. The values of complex formulae are computed via 4-valued truth-tables derived by monotonicity considerations.

Despite its informational flavour, **FDE** is co-NP complete [12, 2] and so an idealized model of how an agent *can* think. A key observation in this work is that a fair amount of idealization is present in the interpretation of the values **t**, **f** and **n**, that presupposes complete information about the set of sources S by an agent a. While the meaning of **b** is "*there is* at least a source assenting to p and at least a source dissenting from p" (which is information empirically accessible to a in that a may actually hold this information without a complete knowledge of S), the meaning of **t**, **f** and **n** involves information of the kind "*there is no* source such that..." (and so requires complete information about the sources in S, which may not be empirically accessible to a at any given time). What if the agent has no such complete knowledge about the sources (e.g., the set of sources is "open")? Inspired by [5] and [10, 11, 3], we address this issue by shifting to *signed* formulae where the signs express *imprecise* values associated with two distinct bipartitions of the standard set of 4 values. These are values such as "**t** *or* **b**", which is implicit in the choice of the set of designated values in the semantics of **FDE**. Thus, we present a proof system which consists of linear operational rules and only two branching structural rules, the latter expressing a *generalized* rule of bivalence. This system naturally leads to defining an infinite hierarchy of tractable depth-bounded approximations to **FDE**. Namely, approximations in which the number of nested applications of the two branching rules is bounded. Further, we show that the resulting hierarchy admits of an intuitive 5-valued non-deterministic semantics.

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