Borel and Borel* Sets in Generalized Descriptive Set Theory

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Generalized descriptive set theory (GDST) aims at developing a higher analogue of classical descriptive set theory in which ω is replaced with an uncountable cardinal κ in all definitions and relevant notions. In the literature on GDST it is often required that $\kappa^{<\kappa} = \kappa$, a condition equivalent to κ regular and $2^{<\kappa} = \kappa$. In contrast, in this talk we use a more general approach and develop in a uniform way the basics of GDST for cardinals κ still satisfying $2^{<\kappa} = \kappa$ but independently of whether they are regular or singular. This allows us to retrieve as a special case the known results for regular κ, but it also uncovers their analogues when κ is singular. We also discuss some new phenomena specifically arising in the singular context (such as the existence of two distinct yet related Borel hierarchies), and obtain some results which are new also in the setup of regular cardinals, such as the existence of unfair Borel* codes for all Borel* sets.

References

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