Set Theory and subalgebras of the Calkin algebra

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In this talk I will discuss the class $\mathbb{E}$ of C^*-algebras that embed into the Calkin algebra $\mathcal{Q}(H)$, and how different set-theoretic axioms influence the nature and properties of such class. By the main result of [1], under the continuum hypothesis the class $\mathbb{E}$ has a clear description: a C^* belongs to $\mathbb{E}$ if and only if its density character is at most $2^{\aleph_0}$, the density character of $\mathcal{Q}(H)$. During this talk I will give an overview of what is known about $\mathbb{E}$ in models of ZFC where the continuum hypothesis fails. More specifically, I will discuss a result from [2], where we prove that the sentence 'Every C^*-algebra of density character less than $2^{\aleph_0}$ embeds into $\mathcal{Q}(H)$' is independent from ZFC $+ 2^{\aleph_0} \geq \aleph_\alpha$, for every $\alpha > 2$, and that such statement is implied by Martin's axiom. Finally, I will expose some of the contents of [3], where I prove that, consistently with ZFC, the class $\mathbb{E}$ is not closed under various classical C^*-algebraic operations, such as countable inductive limits and tensor products.

References

[1] I. Farah and I. Hirshberg and A. Vignati, *The Calkin algebra is $\aleph_1$-universal*, Israel Journal of Mathematics 237 (2020), no. 1, 287–309.

[2] I. Farah and G. Katsimpas and A. Vaccaro, *Embedding C^*-algebras into the Calkin algebra*, International Mathematics Research Notices 2021 (2019), no. 11, 8188–8224.

[3] A. Vaccaro, *Trivial Endomorphisms of the Calkin algebra*, Israel Journal of Mathematics 247 (2021), 873-903.

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