Temporal Epistemic Logic for Awareness with Minimum Delay

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1 Motivation

Logics of awareness appeared in the battle against the problem of logical omniscience in epistemic logic [2]. The problem is that agents know all logical tautologies and the epistemic system contains the epistemic closure under implication. Knowing some formulas, agents also know all logical consequences from them. This gives an overly idealised picture of reasoning agents. An awareness approach solves the problem of logical omniscience by dividing knowledge into implicit and explicit. The last one is not closed under logical inference, therefore we can describe and model resource-bounded agents. This approach was proposed by [3, 5, 1]; it is based on extending standard epistemic logic with awareness operator $A\varphi$ ("be aware of φ"). We will focus on a dynamic approach to awareness logics proposed by [4]. Following the dynamic approach to awareness logic, we have two main goals: 1) construct the awareness logic with temporal relation, 2) consider the particular formal "trigger" for awareness, explaining why an agent becomes aware of something.

In this paper, we present temporal epistemic logic for awareness with minimum delay (TELAMD). Minimum delay means that the agent needs minimum time to become aware of some fact. Thus, TELAMD describes semi-ideal agents, such as robots that do not make mistakes in reasoning, but spend some time on computation. TELAMD formalises the deliberation in a constantly changing flow of information. The basic idea is that agents need time for deliberation to become aware of some facts. But, at the same time, the situation in the world may change, so the agent's awareness should relate to the facts of the past. Where can this model be useful? In situations where the information received is processed before becoming explicit. Let us consider an example.

According to NASA, it takes from 5 to 20 minutes to send radio signals between Mars and Earth (the particular time depends on the planets' position). That is why it makes it difficult to control from Earth in real time, the robots that explore Mars. Let us consider an imaginary autonomous robot Journey-16 that explores the position of the stars and planets from Mars and sends data to Earth. For example, Journey-16 detected a comet and sent its coordinates to Earth. Given that the radio signal from Mars to Earth took 15 minutes, and scientists spent another 2 minutes decrypting the data, the received coordinates of the comet relate to the past, since, during these 17 minutes its position has changed.

2 Language L_{KAXY}

Let Var be a set of atomic propositions, and $\mathcal{A}$ be a finite set of agents. Formulas of the temporal awareness language (L_{KAXY}) are given by the Backus-Naur form: $\varphi, \psi ::= p \mid \neg\varphi \mid$

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$(\varphi \wedge \psi) \mid K_i\varphi \mid A_i\varphi \mid X\varphi \mid Y\varphi$ with $p \in Var$, $i \in \mathcal{A}$. Other Boolean connectives $\vee, \rightarrow, \leftrightarrow$ and dual operators $\hat{K}_i\varphi$, $\hat{X}\varphi$, $\hat{Y}\varphi$ are defined in a standard way: $\hat{K}_i\varphi := \neg K_i\neg\varphi$, $\hat{X}\varphi := \neg X\neg\varphi$, $\hat{Y}\varphi := \neg Y\neg\varphi$. Formulas $K_i\varphi$ are read as "an agent i knows φ implicitly", and formulas $A_i\varphi$ are read as "an agent i is aware of φ". An operator $\Box_i$ is used for explicit knowledge and defined in a following way: $\Box_i\varphi := K_i\varphi \wedge A_i\varphi$.

3 Temporal Awareness Model

A temporal awareness model is a tuple $\mathcal{M} = (W, (\sim_i)_{i\in\mathcal{A}}, \rightsquigarrow, A, V)$ where $(W, (\sim_i)_{i\in\mathcal{A}}, V)$ is a standard epistemic S5-model, $\rightsquigarrow$ is a temporal accessibility relation on W (this relation should be reverse functional), $A_i : W \rightarrow \mathcal{P}(\mathcal{L}_{\mathcal{KAXY}})$ is an awareness function for an agent $i \in \mathcal{A}$.

The truth of a modal formula φ in a pointed model $\mathcal{M}$ is defined as follows: $\mathcal{M}, w \models p \Leftrightarrow w \in V(p)$; $\mathcal{M}, w \models \neg\varphi \Leftrightarrow \mathcal{M}, w \not\models \varphi$; $\mathcal{M}, w \models \varphi \wedge \psi \Leftrightarrow \mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$; $\mathcal{M}, w \models K_i\varphi \Leftrightarrow \forall w'(w \sim_i w' \Rightarrow \mathcal{M}, w' \models \varphi)$; $\mathcal{M}, w \models X\varphi \Leftrightarrow \forall w'(w \rightsquigarrow w' \Rightarrow \mathcal{M}, w' \models \varphi)$; $\mathcal{M}, w \models Y\varphi \Leftrightarrow \forall w'(w' \rightsquigarrow w \Rightarrow \mathcal{M}, w' \models \varphi)$; $\mathcal{M}, w \models A_i\varphi \Leftrightarrow \varphi \in A_i(w)$.

The notions of valid and satisfiable formulas are standard. There are several conditions on awareness function. We use Sub as an abbreviation for set of subformulas. For any $x, y \in W$: (1) $\varphi \in A_i(x), x \rightsquigarrow y \Rightarrow \hat{Y}\varphi \in A_i(y)$; (2) $\varphi \in A_i(x), x \sim_i y \Rightarrow \varphi \in A_i(y)$; (3) $\varphi \in A_i(x) \Leftrightarrow A_i\varphi \in A_i(x)$; (4) $\varphi \in A_i(x) \Leftrightarrow \neg\varphi \in A_i(x)$; (5) $\varphi, \psi \in A_i(x), x \rightsquigarrow y \Rightarrow \varphi \wedge \psi \in A_i(y)$; (6) $\varphi \in A_i(x), x \models K_i\varphi \Rightarrow K_i\varphi \in A_i(x)$; (7) $x \models K_i\varphi \wedge \hat{Y}\neg K_i\varphi, x \rightsquigarrow y \Rightarrow \hat{Y}\varphi \in A_i(y)$; (8) $\varphi \in A_i(x), x \rightsquigarrow y \Rightarrow \psi \in A_i(y)$ for $\psi \in Sub(\varphi)$.

4 Axiomatisation

The following is a sound and complete axiomatization of TELAMD logic: (1) all instances of classical tautologies (2) $S5$ axioms for K_i (3) $X(\varphi \rightarrow \psi) \rightarrow (X\varphi \rightarrow X\psi)$ (4) $Y(\varphi \rightarrow \psi) \rightarrow (Y\varphi \rightarrow Y\psi)$ (5) $\varphi \rightarrow X\hat{Y}\varphi$ (6) $\varphi \rightarrow Y\hat{X}\varphi$ (7) $\hat{Y}\varphi \rightarrow Y\varphi$ (8) $K_iX\varphi \rightarrow XK_i\varphi$ (9) $X\bot \rightarrow K_iX\bot$ (10) $\neg X\bot \rightarrow K_i\neg X\bot$ (11) $Y\bot \rightarrow K_iY\bot$ (12) $A_i\varphi \rightarrow XA_i\varphi$ (13) $A_i\varphi \rightarrow K_iA_i\varphi$ (14) $(\hat{Y}\neg K_i\varphi \wedge K_i\varphi) \rightarrow XA_i\hat{Y}\varphi$ and inference rules (modus ponens and necessitation rules for operators X, Y, K_i).

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