Learning of algebraic structures and Borel equivalence relations

Vittorio Cipriani[1]

Università degli Studi di Udine, Udine, Italia
vittorio.cipriani@uniud.it

Abstract

In this talk, we present some results related to algorithmic learning of algebraic structures. The style of learning we consider is called *learning in the limit* and was Introduced by Gold [6] and Putnam [7] in the 1960s to study how a learner infer objects that are either formal languages or computable functions. The framework we use for objects that are algebraic structures was first introduced in [5] and later refined in [3] and [4]: here a learner receives larger and larger pieces of an arbitrary copy of a computable structure and, at each stage, is required to output a conjecture about the isomorphism type of such a structure. We say that the learning is successful if the conjectures eventually stabilize to a correct guess.

To calibrate the complexity of nonlearnable families, we borrow tools from descriptive set theory, offering a new hierarchy based on reducibility between equivalence relations. To do so, we have defined the notion of *E-learnability*: we say that a family of structures $\mathfrak{K}$ is E-learnable if there is function $\Gamma : 2^\omega \to 2^\omega$ which continuously reduces $\mathsf{LD}(\mathfrak{K})$ to E, where $\mathsf{LD}(\mathfrak{K})$ is the collection of all copies of the structures from $\mathfrak{K}$. We have shown that the paradigm introduced above coincides with E_0-learnability, where E_0 is the eventual agreement on reals. After recalling well-known benchmark Borel equivalence relations, we focus on the learning power of these, differentiating between learnability of finite and countably infinite families.

We then make some considerations on the number of mind-changes needed to learn a given family $\mathfrak{K}$. We show that, for certain families, such a number depends on the height of the poset formed by $\mathfrak{K}$ along with a suitable embedding relation and we give a descriptive set-theoretic interpretation of the mind-change complexity.

This is a joint work with Nikolay Bazhenov and Luca San Mauro: some of the results presented here can be found at [1] and [2].

References

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