Recent semantics for modal systems without replacement of provable equivalents

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Over the last few decades increasing attention has been paid to the problem of developing adequate semantics to interpret systems of modal logic where replacement of provable equivalents (hereafter, RPE) fails. For a given system S, RPE can be formulated as follows:

RPE if $\vdash_S \phi \leftrightarrow \psi$, then $\vdash_S \chi \leftrightarrow \chi^*$, where χ^* is obtained from χ by replacing some occurrences of ϕ with ψ.

Examples of semantics for these systems can be found in [3, 4, 5, 6, 7, 8]. The main motivation at the basis of these investigations is the intuition that there are contexts of reasoning (such as epistemic, doxastic and deontic ones) where modalities are sensitive to *hyperintensional distinctions*: the fact that two propositions are true in the same possible circumstances does not entail that they can be used in an interchangeable way in the scope of a modal operator. Therefore, according to this intuition, RPE should be abandoned in favour of more refined notions of replacement that are not based (or, not uniquely based) on provable equivalence. However, ideas on how to semantically characterize new notions of replacement largely differ.

This presentation will outline two recent solutions developed in the area of normative reasoning [1, 2] and analyse both their expressiveness and some of their fundamental limits. The two solutions are respectively based on the following kinds of relational structures:

- models endowed with a set of semantic contents (*content-based models*);
- models endowed with a relation of propositional pertinence (*pertinence-based models*).

The former models are inspired by a general semantic framework developed in [7], whereas the latter models share some features with the structures proposed in [4].

Given a propositional modal language $\mathcal{L}$ built over a set of propositional variables Var, the definitions of the two kinds of models are as follows:

Definition 1. *A* content-based model *for $\mathcal{L}$ is a tuple $M = \langle W, \mathsf{Cnt}, f, c, V\rangle$ where:*

1. *W is a set of states;*
2. *Cnt is a set of* semantic contents*;*
3. *$f : (\mathcal{L} \times W) \longrightarrow \mathsf{Cnt}$ is a function called* content assignment*;*
4. *$c : W \longrightarrow \wp(\mathsf{Cnt})$ is a function called* necessity assignment*;*
5. *$V : \mathtt{Var} \longrightarrow \wp(W)$ is a valuation function.*

The truth-value of formulas containing occurrences of modal operators at a state w of a model M is determined by the interaction between functions f and c. In particular, $M, w \models \Box\phi$ iff $f(\phi, w) \in c(w)$. Thus, whenever $f(\psi, w) \neq f(\chi, w)$ it is possible to have $M, w \models \Box\psi$ and $M, w \not\models \Box\chi$.

Definition 2. *A* pertinence-based model *for $\mathcal{L}$ is a tuple $M = \langle W, R, C, V \rangle$ where:*

- *W is a set of states;*
- *$R \subseteq \mathcal{L} \times \mathcal{L} \times W$ is a relation called* propositional pertinence*;*
- *$C \subseteq R$ is a relation called* necessity distribution*;*
- *$V : \mathtt{Var} \longrightarrow \wp(W)$ is a valuation function.*

The truth-value of formulas containing occurrences of modal operators at a state w of a model M is determined by the interaction between relations R and C. In particular, $M, w \models \Box\phi$ iff $(\phi, \phi, w) \in C$. Thus, whenever $\psi \neq \chi$ it is possible to have $M, w \models \Box\psi$ and $M, w \not\models \Box\chi$.

Extending some results in [2], it will be proven that each content-based model can be transformed into an equivalent pertinence-based model satisfying certain properties, and vice versa. This means that, under certain translation functions, the expressiveness of the two semantics is ultimately the same. However, the final part of the presentation will illustrate some of the expressive limits of the two semantics with respect to the representation of hyperintensional reasoning and suggest that an additional refinement of these structures is needed.

References

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