

Nonlinearity and illfoundedness in the hierarchy of consistency strength and the question of naturality

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Set theorists and philosophers of mathematics often point to a mystery in the foundations of mathematics, namely, that our best and strongest mathematical theories seem to be linearly ordered and indeed well-ordered by consistency strength. Why should it be? The phenomenon is thought to carry profound significance for the philosophy of mathematics, perhaps pointing us toward the ultimately correct mathematical theories, the “one road upward.” And yet, we know as a purely formal matter that the hierarchy of consistency strength is not well-ordered. It is ill-founded, densely ordered, and nonlinear. The statements usually used to illustrate these features, however, are often dismissed as unnatural or as Gödelian trickery. In this talk, I aim to rebut that criticism by presenting a variety of natural hypotheses that reveal ill-foundedness in consistency strength, density in the hierarchy of consistency strength, and incomparability in consistency strength.