Presentazione

Questo è un "e-Proceedings" (electronic Proceedings), relativo ai lavori della *Giornata della FIMA*, la Federazione Italiana di Matematica Applicata, svoltasi il 27 Ottobre 2017, nell'aula Marconi della Sede del CNR, a Roma.

Si tratta delle slides delle presentazioni di 10 esperti, dei quali 2 proposti da ciascuna delle 5 associazioni scientifiche che costituiscono al momento attuale la FIMA, e cioè **AILA** (Associazione Italiana di Logica e sue Applicazioni), **AIRO** (Associazione Italiana di Ricerca Operativa), **AMASES** (Associazione per la Matematica Applicata alle Scienze Economiche e Sociali), **AIMETA** (Associazione Italiana di Meccanica Teorica e Applicata) e **SIMAI** (Società Italiana di Matematica Applicata e Industriale).

Nel corso della giornata di lavoro, i 5 Presidenti (o loro delgati) hanno firmato, a titolo di Convenzione, uno Statuto che ufficializza la Federazione.

I 10 conferenzieri sono stati scelti, a titolo di Comitato Scientifico, dai 5 Presidenti delle associazioni suddette, rispettivamente Antonio Di Nola (AILA), Daniele Vigo (AIRO), Bruno Viscolani (AMASES), Paolo Luchini (AIMETA) e Luca Formaggia (SIMAI), oltre al Presidente uscente dell'AILA, Carlo Toffalori, a Sandra Carillo (facente le funzioni del Presidente in carica dell'AIMETA), e dal sottoscritto.

Renato Spigler

Roma, 3 Novembre 2017

FEDERAZIONE ITALIANA DI MATEMATICA APPLICATA

STATUTO

Art. 1 (Denominazione, sede e durata)

Il nome della Federazione è FEDERAZIONE ITALIANA DI MATEMATICA APPLICATA (FIMA)

La Federazione riunisce Associazioni scientifiche italiane inerenti la Matematica Applicata.

La Federazione è una organizzazione scientifica senza fini di lucro e la sua durata è illimitata.

Essa ha la sede sociale in Roma presso l'Istituto per le Applicazioni del Calcolo (IAC) del CNR.

Art. 2 (Scopi)

La Federazione ha come scopi:

- riunire, promuovere ed incentivare lo scambio scientifico, tecnico ed applicativo nel campo della Matematica Applicata;

- favorire lo scambio di informazioni e le relazioni fra i vari Enti che si occupano di Matematica Applicata, promuovere la stipula di convenzioni con tali Enti e la partecipazione a organismi internazionali;

- promuovere congressi, seminari e workshop;

- favorire la ricerca scientifica e la costituzione di gruppi di ricerca;

- incoraggiare e stimolare la diffusione e l'insegnamento della Matematica Applicata;

- promuovere specifiche pubblicazioni di carattere scientifico e divulgativo della materia.

Art 3 (Categorie di Associati, autonomia e modalità di adesione)

Gli Associati della Federazione sono singole Associazioni che condividono gli scopi di cui al precedente articolo e che hanno sottoscritto l'atto costitutivo o che hanno successivamente presentato domanda di adesione, approvata come sotto specificato. Esse sono rappresentate dai loro Presidenti pro-tempore o da loro delegati.

All'atto della sua costituzione, nell'anno 2004, la Federazione riuni' le tre associazioni scientifiche seguenti: AIRO, AMASES, SIMAI. Dall'anno 2017 aderiscono ulteriormente le associazioni scientifiche AILA e AIMETA.

La Federazione garantisce la piena autonomia dei singoli Associati nel perseguimento dei propri fini istituzionali.

Art. 4 (Organi)

Sono organi della Federazione:

- Il Consiglio Direttivo
- Il Presidente

Art. 5 (Composizione e Attribuzioni del Consiglio Direttivo)

Il Consiglio Direttivo è costituito dal Presidente e dai Presidenti pro-tempore delle singole Associazioni afferenti alla Federazione (o loro delegati).

Esso delibera all'unanimità circa l'ammissione di nuovi Associati che abbiano presentato domanda di adesione ai sensi dell'Art. 3. La cessazione degli Associati avviene a seguito di dimissioni o per decisione presa all'unanimità dai restanti Associati.

Il Consiglio Direttivo è responsabile della gestione economica della Federazione.

Il Consiglio Direttivo coordina l'attività della Federazione nel perseguimento dei propri scopi istituzionali.

L'eventuale scioglimento della Federazione deve essere deliberato all'unanimità dal Consiglio Direttivo e l'atto di scioglimento deve essere raccolto in sede notarile. In caso di riduzione degli Associati ad uno solo, sarà esso a deliberare al più presto con le forme sue proprie.

Art. 6 (Nomina e Attribuzioni del Presidente)

La carica di Presidente della Federazione, che viene eletto dal Consiglio Direttivo fra i Presidenti delle Associazioni facenti parte della FIMA eventualmente sostituiti dai corrispondenti delegati, ha la durata di due anni e non puo' essere coperta consecutivamente da membri della stessa Associazione. Il Presidente presiede le riunioni del Consiglio Direttivo, ha la legale rappresentanza della Federazione di fronte a terzi ed esercita ogni altra attribuzione inerente alla carica, a norma di legge.

Art. 7 (Impegni degli Associati)

Gli Associati si impegnano al rispetto del presente Statuto e delle deliberazioni e regolamenti emanati dagli organi della Federazione.

Gli Organi deliberanti degli Associati stabiliscono un accordo di reciprocità in base al quale i soci di qualsiasi tipologia di ciascuno dei predetti Associati hanno facoltà di iscriversi a Convegni, Congressi, Giornate di lavoro e simili organizzati dagli altri Associati con le medesime condizioni e facilitazioni riservate ai soci dell'Associato che organizza.

Art. 8 (Risorse della Federazione)

Le risorse della Federazione sono costituite da contributi a qualunque titolo ricevuti direttamente, nonche' da quanto deciso di attribuirle da parte dei Consigli Direttivi delle Associazioni afferenti.

In caso di scioglimento, il Consiglio Direttivo deliberera' sulla destinazione delle risorse residue, tenendo conto degli oneri latenti da fronteggiare.

Art. 9 (Modifiche di Statuto)

Il presente Statuto può essere modificato dagli Associati, con decisione unanime.

Art. 10 (Norma conclusiva)

Per tutto quanto non previsto dal presente Statuto si applicano le disposizioni degli Artt. 36 e segg. del Codice Civile e delle leggi vigenti in materia.

Per AILA, il Presidente, Prof. Antonio Di Nola:

Per AIRO, il Presidente, Prof. Daniele Vigo:

Per AIMETA, *per* il Presidente, Prof. Paolo Luchini, *la* Prof.ssa Sandra Carillo:

Per AMASES, il Presidente, Prof. Bruno Viscolani:

Per SIMAI, il Presidente, Prof. Luca Formaggia:







GIORNATA DELLA FIMA FEDERAZIONE ITALIANA DI MATEMATICA APPLICATA



Venerdi 27 Ottobre 2017 ore 10.00 AULA MARCONI – CNR, PIAZZALE ALDO MORO, 7 ROMA



Speakers

SIMAI Renzo Ricca, Università di Milano Bicocca Antonio De Simone, SISSA, Trieste AILA Silvio Ghilardi, Università di Milano Hykel Hosni, Università di Milano

AMASES

Emanuele Borgonovo, Università Bocconi, Milano **Michele Tumminello**, Università di Palermo

AIRO Fabio Schoen Università di Firenze Laura Palagi, Sapienza Università di Roma

AIMETA

Elena Bonetti, Università di Milano Giuseppe Piccardo, Università di Genova

In collaborazione con

- Sezione di Matematica dell'UNIVERSITÀ TELEMATICA INTERNAZIONALE UNINETTUNO
- Dipartimento di Matematica e Fisica dell'Università RomaTre



UNIVERSITÀ TELEMATICA INTERNAZIONALE UNINETTUNO

PROGRAMMA GIORNATA della FIMA

27 Ottobre 2017, aula Marconi - CNR, Roma

h. 10-10.30: Fabio Schoen Università di Firenze [**AIRO**], "Optimization: a unifying framework for applied problem solving"

h. 10.30-11: Renzo Ricca, Milano Bicocca [**SIMAI**], "Knot polynomials as a new tool for turbulence research".

h. 11-11.30: Silvio Ghilardi, Università di Milano, Dip. di Matematica [**AILA**], "Logica, model checking e verifica formale".

h. 11.30-12: Giuseppe Piccardo, Università di Genova, Dip. Ing. Civile e Amb. [AIMETA], "Internal resonances in aeroelastic oscillations".

h. 12-12.30: Emanuele Borgonovo, Bocconi, Milano [**AMASES**], "Big Data and business analytics: a critical overview".

h.12.30-13: Elena Bonetti, Università di Milano [**AIMETA**], "A nonlinear model for marble sulphation including surface rugosity: analytical and numerical results".

h. 13.00: firma dei Presidenti

PAUSA LUNCH

h. 14.30-15: Antonio De Simone, SISSA, TS [**SIMAI**], "Biological and bio-inspired motility at microscopic scales".

h. 15-15.30: Laura Palagi, Sapienza Università di Roma [AIRO], "Optimization and machine learning".

h. 15.30-16: Michele Tumminello, Università di Palermo [**AMASES**], "Big Data and Networks for Fraud Detection in the Insurance Sector"

h. 16-16.30: Hykel Hosni, Università di Milano, Dip. di Filosofia [AILA], "Logica nelle decisioni in condizione di incertezza".

Optimization: a unifying framework for applied problem solving

Fabio Schoen



ASSOCIAZIONE ITALIANA DI RICERCA OPERATIVA

Outline

- Foreword
- Optimization everywhere . . .
- Brief introduction to the Global Optimization Laboratory Univ. of Florence

http://gol.dinfo.unifi.it

Part I

Foreword

(personal view): Milestones in Applied Optimization

(personal view): Milestones in Applied Optimization



(personal view): Milestones in Applied Optimization: Didone

Devenere locos ubi nunc ingentia cernis Moenia sergentemque novae Karthaginis arcem, mercatique, solum, facti de nomine Byrsam, taurino quantum possent circumdare tergo.

Giunsero in questi luoghi, ov'or vedrai sorger la gran cittade e l'alta rocca della nuova Carthago, che dal fatto Birsa nomassi, per l'astuta merce che, per fondarla, fèr di tanto sito quanto cerchiar di bue potesse un tergo

Virgilio, Eneide

(personal view): Milestones in Applied Optimization: G.B. Dantzig

In the summer of 1947, when I began to work on the simplex method for solving linear programs, the first idea that occurred to me is one that would occur to any trained mathematician, namely the idea of step-by-step descent (with respect to the objective function) along edges of the convex polyhedral set from one vertex to an adjacent one. I rejected this algorithm outright on intuitive grounds-it had to be inefficient because it proposed to solve the problem by wandering along some path of outside edges until the optimal vertex was reached. I therefore began to look for other methods that gave more promise of being efficient, such as those that went directly through the interior [4]. George G.B. Dantzig

(personal view): Milestones in Applied Optimization: Sir Francis Galton



(personal view): Milestones in Applied Optimization: McCulloch and Pitts

BULLETIN OF MATHEMATICAL BIOPHYSICS VOLUME 5, 1943

A LOGICAL CALCULUS OF THE IDEAS IMMANENT IN NERVOUS ACTIVITY

WARREN S. MCCULLOCH AND WALTER PITTS

FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE, DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INSTITUTE, AND THE UNIVERSITY OF CHICAGO

Because of the "all-or-none" character of nervous activity, neural events and the relations among them can be treated by means of propositional logic. It is found that the behavior of every net can be described in these terms, with the addition of more complicated logical means for nets containing circles; and that for any logical expression satisfying certain conditions, one can find a net behaving in the fashion it describes. It is shown that many particular choices among possible neurophysiologiing under one assumption, there is another net which behaves under the other and gives the same results, although perhaps not in the same time. Various applications of the calculus are discussed.

Part II

Applied Optimization:

Part II

Applied Optimization: INFORMS Edelman Prize



UPS Optimizes Delivery Routes Driver's Solution



ORION Solution



KKT srl, now a Verizon company





360i Generates Nearly \$1 Billion in Revenue for Internet Paid-Search Clients

Optimization models for: forecasting, budget allocation, bidding





Bank of NY Mellon Optimization Reduces Intraday Credit Risk by \$1.4 Trillion

A set of integrated mixed-integer programming models to solve collateral-management challenges involving short-term secured loans.

Minimize intraday credit exposure and the liquidity usage of its clients.

Each day rebalancing and continuous portfolio optimization employed to process \$1.4 trillion of client collateral.

Reduced intraday credit risk by more than 97%



Operations Research Transforms the Scheduling of Chilean Soccer Leagues and South American World Cup Qualifiers



Optimize cost & audience taking constraints into account

Scheduling the Italian Volley Championship

Video on YouTube: https://youtu.be/9A35btfoGlo



2017 Edelman Prize winner and finalists



Barco BARCo

(visualization and image processing solutions)

2017 Edelman Prize winner and finalists

BHP Billiton bipbiliton
(discovery, acquisition, development and marketing of natural resources, e.g., mineral extraction, petroleum)
General Electric (mission: build, power, cure, move, and connect the world)

New York City Department of Transportation

http://gol.dinfo.unifi.it

Part III

GOL: Global Optimization Laboratory

GOL: who we are

2 professor, one researcher, 7 PhD students, 1 post-doc Many students. . .

GOL - Global Optimization Laboratory

Recent Research Projects (EU, Tuscany Region):

- (2016-) LINFA Logistica Intelligente del Farmaco: optimal decisions in drug replenishment at hospital wards
- (2015-) SiiMobility Supporto all'interoperabilità integrata per i servizi ai cittadini e alla pubblica amministrazione: optimal path planning for multi-modal multi-objective paths (foot, car, bike, wheelchair, ...)
- (2014) PRESTIGE Progetto di RicErca e Sviluppo di servizi Turistici Innovativi basati su architettura abilitante attuato da un Gruppo di opEratori sinergici: optimization of touristic tours
- (2013-2015) SMARTY Smart Transportation for sustainable city: optimal relocation in bike sharing systems, optimal traffic equilibrium
- (2013) MIDA: Machine Intelligence for Diagnosys Automation: optimal prediction of machine failures from oil analyses

GOL - Global Optimization Laboratory

Recent Research Projects (private companies):

- (2016-2017): ENEL: modelli di previsione della produzione fisica e dell'offerta a mercato di impianti rinnovabili machine learning for optimal prediction in the energy market
- (2014-2016): Kering / Gucci: Demand forecasting, optimal inventory planning, optimal replenishment decisions
- ► (2015-2016) KKT: optimization for vehicle routing
- (2016-2017) Lega Volley: Optimal scheduling of the Italian League Volley Tournament

Our future

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The Sexiest Job of the 21st Century: Data Analyst

Chris Morris, Special to CNBC.com Published 1:00 PM ET Wed, 5 June 2013 | Updated 4:50 PM ET Wed, 5 June 2013

SCNBC



Photo: Biddlboo | Getty Images

Looking for a career change or a college major that's all but guaranteed to result in a hefty salary with copious benefits? Big data may not seem the obvious choice, but it could be your best.

With more and more companies using big data, the demand for data

Best jobs ranking 2017 - . CAREERCAST

| | Statistician | | |
|-------------------|---|----------------------------|-------------------------|
| | Overall Rating: 1/199 | | Median Salary: \$80,110 |
| | Work Environment | Stress | Projected Growth |
| | Very Good | Very Low | Very Good |
| | 4/199 | 39/199 | 3/199 |
| | | View Raw Scores | |
| | | | |
| | | Search Statistician Jobs (| 9 |
| 2 | Medical Services Ma | nager | |
| | Overall Rating: 2/199 | | Median Salary: \$94,500 |
| | Work Environment | Stress | Projected Growth |
| | Very Good | Very Low | Very Good |
| | 23/199 | 17/199 | 32/199 |
| | | View Raw Scores | |
| | | | |
| | Search Medical Services Manager lobs | | |
| | | | |
| | Operations Research | n Analyst | |
| | Overall Rating: 3/199 | | Median Salary: \$79,200 |
| | Work Environment | Stress | Projected Growth |
| | Very Good | Very Low | Very Good |
| | 35/199 | 10/199 | 6/199 |
| | | View Raw Scores | |
| The second second | | | |
| | | | |
| | Search Operations Research Analyst Jobs 🕤 | | |

Optimization: a unifying framework for applied problem solving

Fabio Schoen



ASSOCIAZIONE ITALIANA DI RICERCA OPERATIVA

Rome – 27 October, 2017

Knot polynomials as a new tool for turbulence research

RENZO L. RICCA

Department of Mathematics & Applications, U. Milano-Bicocca, Italy renzo.ricca@unimib.it

Objectives

- Determine relationships between structural complexity of physical knots and energy;
- Quantify energy/helicity transfers in dynamical systems.



- Knot polynomials as new physical invariants to quantify topological complexity;
- Extend and apply new topological techniques to study complex systems.



Coherent structures



Leonardo da Vinci (Water Studies 1506)





Vorticity localization in quantum fluids



Miyazaki et al. (*Physica D* 2009)



Laurie & Baggaley (JLTP 2015)
Modeling vortex tangles by filaments

homogeneous incompressible inviscid fluid in \mathbb{R}^3 : u = u(X,t) $\begin{cases} \nabla \cdot u = 0 & \text{in } \mathbb{R}^3 \\ u = 0 & \text{as } X \to \infty \end{cases}$

• Vortex line χ : vorticity: $\omega = \nabla \times u$,

$$\omega = \overline{\omega}_0 \hat{t}$$
, $\overline{\omega}_0$ = constant;

Vortex tangle:
$$\mathcal{T}' = \bigcup_i \chi_i \quad i = 1, ..., N$$
.

• Kinetic energy:
$$E(\mathcal{T}') = \int_{\Omega} \boldsymbol{u} \cdot (\boldsymbol{X} \times \boldsymbol{\omega}) d^{3}\boldsymbol{X}$$



from Lamb (1932), we have:
$$E(\mathcal{T}') \approx \frac{\Gamma^2}{4\pi} \sum_{ij} \iint_{\chi_i \chi_j} \frac{t_i \cdot t_j}{|X_i - X_j|} \, \mathrm{d}s_i \mathrm{d}s_j$$
.

• Total length of vortex tangle given by $L(\mathcal{T}') = \sum_{i} \int_{\gamma_i} \hat{t}_i \, \mathrm{d}s_i$.

Kinetic helicity and linking numbers

• Kinetic helicity:

$$H(\mathcal{T}) = \int_{\mathcal{T}} \boldsymbol{u} \cdot \boldsymbol{\omega} \, \mathrm{d}^{3} \boldsymbol{X} = \Gamma \sum_{i} \int_{\chi_{i}} \boldsymbol{u} \cdot \mathrm{d} \boldsymbol{l} \, .$$

• Theorem (Moffatt 1969; Moffatt & Ricca 1992). Let T be an essential physical link in an ideal fluid. Then, we have:

$$H(\mathcal{T}) = \Gamma^2 \left(\sum_i SL_i + \sum_{i \neq j} Lk_{ij} \right) \begin{cases} SL_i = SL(\chi_i) \text{ self-linking number} \\ Lk_{ij} = Lk(\chi_i, \chi_j) \text{ linking number} \end{cases}$$

• Self-linking number (Călugăreanu-White invariant):

Consider the ribbon $\Re(\chi,\chi^*)$; then $SL = \lim_{\epsilon \to 0} Lk(\chi,\chi^*)$, where

$$SL = Wr(\chi) + Tw(\chi, \chi^*)$$

writhing number: $Wr(\chi)$

total twist number:

$$Tw(\chi,\chi^*)$$



Tackling structural complexity by knot polynomials

• Helicity and linking number limitations:

(i)
$$H(\mathcal{T}) = f(SL_i, Lk_{ij}; \Gamma)$$

(ii)
$$Lk_{ij} = 0$$
, $\sum_{i \neq j} Lk_{ij} = 0$.

• HOMFLYPT polynomial $P(\chi) = P_{\chi}(a,z)$: (P.1) P(O) = 1(P.2) $aP(\chi) - a^{-1}P(\chi) = zP(\chi)$



$$\Pi_i \xrightarrow{-1} \xrightarrow{+1}$$

$$\bigcup_{U_1} \sim \bigcup_{\gamma_+} \sim \bigcup_{\gamma_-} P(O) = P(\gamma_+) = P(\gamma_-) = 1$$

$$\bigcup_{\gamma_+} \qquad \bigvee_{\gamma_-} \qquad \bigcup_{U_2} \rightarrow P(U_2) = \frac{a - a^{-1}}{z}$$

P.2: (

HOMFLYPT polynomial from self-linking

• Theorem (Liu & Ricca, JFM 2015). If χ denotes a vortex knot of helicity $H = H(\chi)$, then

$$e^{H(\chi)}=e^{SL(\chi)}$$
 ,

appropriately rescaled, satisfies (with a plausible statistical hypothesis) the skein relations of the HOMFLYPT polynomial $P(\chi) = P_{\chi}(a,z)$.

• HOMFLYPT variables in terms of writhe and twist:

$$a = f(Tw)$$
, $z = g(Wr)$.

Hence, from the HOMFLYPT (P.2) skein relation, we have:

$$aP(\mathcal{N}) - a^{-1}P(\mathcal{N}) = zP(\mathcal{N}) \quad \Longleftrightarrow \quad [f(Tw)] = g(Wr) \; .$$

• Reduction of HOMFLYPT $P_{\chi}(a,z)$ to Jones $V_{\chi}(t)$:

t = h(a,z) : $W_{r} \propto T_{W}$ \iff knots and links are "framed".

Vortex trefoil cascade process in water (Kleckner & Irvine 2013)









t = 2



T(2,1)





t = 3

Quantum vortex link cascade in BECs (Zuccher & Ricca, PRE 2017)



Torus knots & links cascade modeling

Consider the cascade process:

(i)
$$C \rightarrow O \rightarrow O$$

(ii) ... $T(2,7) \xrightarrow{T(2,6)} T(2,5) \xrightarrow{T(2,4)} T(2,3) \xrightarrow{T(2,2)} T(2,1) \xrightarrow{T(2,0)} T(2,n) \xrightarrow{T(2$

• Theorem (Liu & Ricca, Nature Sci Rep 2016). HOMFLYPT computation of $P_{T(2,n)}$ generates, for decreasing n, a monotonically decreasing sequence of numerical values given by

 $P_{T(2,3+q)} = A_q(\tau, \omega) P_{T(2,3)} + B_q(\tau, \omega) P_{T(2,2)} \quad (q \in \mathbb{N}) ,$ where $A_q(\tau, \omega)$ and $B_q(\tau, \omega)$ are known functions of τ and ω , with initial conditions $P_{T(2,3)}$ and $P_{T(2,2)}$.

Vortex trefoil cascade process in water (Kleckner & Irvine 2013)



t = 1

















Vortex link cascade in BECs (Zuccher & Ricca, PRE 2017)



HOMFLYPT quantifies topological complexity



• Comparative analysis: HOMFLYPT versus other knot polynomials

| Numerical values for torus knots and <i>co-oriented</i> torus links ($Wr = Tw = 1/2$) | | | | | | | | | | | |
|---|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | T(2,10) | T(2,9) | T(2,8) | T(2,7) | T(2,6) | T(2,5) | T(2,4) | T(2,3) | T(2,2) | T(2,1) | T(2,0) |
| HOMFLYPT: $a = e^{1/4}, k = e^{1/2}$ | 8.52 | 6.64 | 5.17 | 4.03 | 3.13 | 2.44 | 1.89 | 1.50 | 1.11 | 1 | 0.48 |
| Jones: $t = e^{-1}$ | -0.01 | 0.02 | -0.03 | 0.05 | -0.09 | 0.15 | -0.25 | 0.40 | -0.69 | 1 | -2.26 |
| Alexander-Conway: $\varsigma = e^{-1}$ | -65.81 | 39.92 | -24.20 | 14.70 | -8.88 | 5.44 | -3.22 | 2.08 | -1.04 | 1 | |

Conclusions and outlook

- Adapted HOMFLYPT is the best quantifier of cascade processes:
 - P_K provides monotonic behavior consistently;
 - numerical values more robust and reliable markers for diagnostics.
- Same cascade in recombinant DNA plasmids (Shimokawa et al. 2013):



• Optimal path to cascade?

$$P_{T(2,5)} = 2.44 \rightarrow P_{T(2,4)} = 1.89$$

$$P_{T(2,5)} = 3.13$$

$$P_{T(2,5)} = 2.44 \rightarrow P_{T(2,4)} = 1.89$$

$$P_{T(2,3)} = 1.50$$

$$P_{3_1\#3_1} = 2.26 \rightarrow P_{3_1\#2_{2,1}} = 1.66$$

LOGICA MODEL CHECKING VERIFICA FORMALE

Silvio Ghilardi

silvio.ghilardi@unimi.it

http://users.mat.unimi.it/users/ghilardi/









Revolutionized computer program analysis with temporal logic

AMIR PNUELI

acm





While working at the department of computer science at Tel Aviv University, <u>Amir Pnueli</u> became deeply involved in logics and deductive methods, and particularly in the work of Arthur Prior. Pnueli became the first to realize the potential implications of applying Prior's "tense logic" to computer programs. **Pneuli's 1977 seminal paper, "The Temporal Logic of Programs," revolutionized the way computer programs are analyzed**. At the time, **practical program verification was widely considered to be hopeless, but Pnueli's paper introduced the notion of reasoning about programs as execution paths, breathing new life into the field of program verification. For this visionary work combining temporal logic with computing science, Pnueli received the 1996** ACM A.M. Turing Award.





EDMUND M. CLARKE, E. ALLEN EMERSON, JOSEPH SIFAKIS Model Checking: An Automated Quality Assurance Method





In **1981**, seminal papers by Edmund M. Clarke and E. Allen Emerson working in the USA, and by Joseph Sifakis working independently in France, founded an automatic quality assurance method, dubbed Model Checking. This method provides an **algorithmic means of verifying whether or not an abstract model representing a system or design, satisfies a formal specification** expressed in temporal logic. The progression of Model Checking technology to the point where it can be successfully used for very complex systems has required the development of sophisticated means of coping with extremely large state spaces, and substantive increases in expressiveness. The laureates work on these topics has further engendered a very large international research community investigating such topics. Consequently, many major hardware and software companies are now using Model Checking in practice. Applications include formal verification of VLSI circuits, communication protocols, and embedded systems.





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model checking, An Addonated Quanty Assurance method





In 1981, seminal papers by Edmund M. Clarke and E. Allen Emerson working in the USA, and by Joseph Sifakis working independently in France, founded an automatic quality assurance method, dubbed Model Checking. This method provides an algorithmic means of verifying whether or not an abstract model representing a system or design, satisfies a formal specification expressed in temporal logic. The progression of Model Checking technology to the point where it can be successfully used for very complex systems has required the **development of sophisticated means of coping with extremely large state spaces**, and substantive increases in expressiveness. The laureates work on these topics has further engendered a very large international research community investigating such topics. Consequently, many major hardware and software companies are now using Model Checking in practice. Applications include formal verification of VLSI circuits, communication protocols, and embedded systems.

model éncening. An Adomated Quanty Assurance method





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MODEL CHECKING: MAIN INGREDIENTS

Temporal logic

To express a formal specification/property

- Transition systems
 - To describe an abstract model of a system/design
- Algorithms
 - To verify or not that an abstract model satisfies a formal specification
 - By exploring all possible execution paths



MODEL CHECKING: THE PROBLEM

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MODEL CHECKING: PROBLEM VARIANTS

There are many different variants according to different (classes of)



Need to develop scalable algorithms to check for satisfaction



MODEL CHECKING: PROBLEM VARIANTS (CONT'D)

- System models: design choices
 - State-based vs Event-based

• ...

- Interleaving vs True Concurrency
- Synchronous vs Asynchronous interaction





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Choices in the literature yield

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- Petri nets
- I/O Automata
- Reactive modules
- Logical formulae

MODEL CHECKING: PROBLEM VARIANTS (CONT'D)

- System models: design choices
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...
 Motivation: ease of modeling

Choices in the literature yield

- CSP
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MODEL CHECKING: PROBLEM VARIANTS (CONT'D)

- System properties: design choices
 - Automata vs Logic
 - Linear vs Branching
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 - Safety vs Liveness
- **Liveness** = something *good* will *eventually* happen

Safety = something bad will never happen

- Don't know when
- Propositional vs First-Order



M = S

MODEL CHECKING: PROBLEM VARIANTS (CONT'D)

- System properties: design choices
 - Automata vs Logic
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 - Safety =
 - witness Propos

- Safety = something bad will never happen
- **Liveness** = something *good* will *eventually* happen
- Don't know when

 Liveness = properties whose violation has no finite witness



M = S

MODEL CHECKING IN SHORT

- Initially, MC was for finite state systems
 - Widely adopted in the HW industry
 - First workable technique for automated verification
- Afterwards, MC modified and extended in many different directions
 - Real-time & hybrid systems
 - SW (abstraction & refinement)
 - Petri nets
 - Infinite-state systems
 - Parametric and parametrized systems
 - Distributed systems
 - MC = Algorithm (automated) verification
- Symbolic methods increasingly adopted



- We call I the set of initial states, U the set of undesired states, τ the transition relation (the system moves from a state s to any state s' such that the condition $\tau(s, s')$ is satisfied).
- \blacksquare We iteratively compute the preimage of U applying backward τ





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- ... until we find an intersection with the set of initial states...



Safety Problems (informal description - very basic schema)

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- \blacksquare We iteratively compute the preimage of U applying backward τ
- ... until we find an intersection with the set of initial states...
- ... or a (global) fix-point.


In the former case the system contains a bug.

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In the latter case, a safety certification is reached (notice that the complement of the 'red' region is an invariant for the system - i.e. it is a set of states encompassing initial states and closed under the application of transitions).

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- $U_{n+1} := \exists \underline{a}' \ (\tau(\underline{a}, \underline{a}') \land U_n(\underline{a}')).$

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All what we need is to generate automatically formulae to be proved valid/unsatisfiable: these are the the so-called **proof obbligations**. Finally, we need the appropriate technology to 'discharge' such proof obbligations. This technology is supplied by the so-called **SMT-solvers**.

Reduce intersection and fix-point test to satisfiability problems:

• Intersection test: is $I \wedge U_n$ satisfiable?



Reduce intersection and fix-point test to satisfiability problems:

- Intersection test: is $I \wedge U_n$ satisfiable?
- Fix-point test: is $U_{n+1} \rightarrow U_n$ valid?
- ... or equivalently: is $U_{n+1} \wedge \neg U_n$ unsatisfiable?



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- Refinements are guided by counterexamples (i.e. by the analysis of spurious traces arising).
- There are plenty of techniques (we won't analyze here): acceleration, predicate abstraction, interpolation, property directed reachability,...
- In all cases, the model checker needs to interact with a solver discharging proof obbligations.

Architecture

- **client** Reachability analysis via generation of proof obbligations
- **server** SMT-solver: decides the (un)satisfiability of proof obbligations.



SAT

- SAT problem
 - Is there an assignment that makes a given formula true?
 - Assignment to Boolean variables
- First NP-complete problem [Cook theorem, 1971]
- In practice, state-of-the-art SAT solvers can often solve problems problems with millions of variables/constrai
- Limited input language
 - Propositional logic
 - Difficulties in modeling



Extension of Davis-Putnam-Loveland-Logemann (DPPL) algorithm



9/28/1



THE SAT ASSOCIATION

DPLL, CDCL, SLS, GB, SMT, BDD, AIG, MAX-SAT, Equiv Check, Model Check, more ...

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SAT

Association

satisfiability competition

reference satlive! smtlib qbflib



Purpose

The aim of the Association is to promote science and research, in particular with regard to the Satisfiability Problem and related areas such as Formal Verification and other applications of SAT, Proof Complexity, Serial and Parallel SAT Solvers, Satisfiability Modulo Theories, Quantified Boolean Formulas, SAT Algorithms, MAX-SAT, MUS Extraction, and SAT Encodings. This is realized by overseeing the organization and location of the series of annual conferences known as the *International Conference on the Theory and Applications of Satisfiability Testing*, promoting research in the design and analysis of algorithms for Satisfiability that should significantly impact one or more practical applications, publishing the online version of the *Journal on Satisfiability, Boolean Modeling, and Computation* (known as JSAT), and representing the SAT community in other scientific forums.



SMT

- SMT problem
 - Is there an assignment that makes a given formula true?
 - Assignment to First-order variables
- NP-hard problem
- In practice, state-of-the-art SMT solvers can often solve problems with millions of variables/constraint
 - Typically formulae are quantifier-free, although...
 - ... recently there have been advances to cope with controlled \mathfrak{q}_{-}
- Richer input language
 - First-order logic
 - Much easier to specify models and properties (in particular of SW systems)



Lazy SMT solver architecture combining DPLL with theory solvers



9/28/1







• the core system of an SMT solver takes as input a quantifier free formula in a combined theory, like

 $(j < i \land a' = wr(a, j, a[j] + 1) \land j' = j + 1 \land pc' = pc) \lor$ $(j \ge i \land a' = a \land pc' = L)$

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- if an inconsistency is found, lemmata are learned and the control goes back to the DPLL engine for backtracking.

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- fragments of higher order logic also arise (e.g. cardinality constraints);
- research is very active in the area, to explore decidability limits and to determine complexities.

• some of these groups maintain originally developed tools;

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- in the next slides, I shall concentrate on some contributions by my group in Milano.

MCMT

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MCMT web site

MCMT: Model Checker Modulo Theories

Last Release: version 2.5.2 -- Last Update: 18/6/2017

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- Instructions
- Publications
- Experiments
- Related Tools
- Aknowledgments

MT is a model checker for infinite state systems based on the integration of Satisfiability Modulo Theories (SMT) solving and

MCMT is a model checker for infinite state systems based on the integration of <u>Satisfiability Modulo Theories</u> (SMT) solving and backward reachability. In its actual implementation, MCMT is capable of model checking invariant (safety) properties of a large class of infinite state systems called array-based systems.

Download

MCMT is distributed "as is" without warranty of any kind; it is free for non-commercial use. It is possible to download the executables for Linux (both 32/64 bit) and for Mac OS X together with some examples. The executables for the old versions 2.0 and 1.1.1 are still here and here.

Experiments

We run MCMT on various examples: parametrized mutual exclusion and cache coherence protocols, timed systems, imperative programs, etc. These experiments show the great flexibility of the tool. All the files for these examples are included in the distribution (in each file, a brief description of the source and the meaning of the example is supplied). Table with experimental statistics (concerning the old version 1.0) are available <u>here</u>. Further case studies have been analyzed:

- Parameterized Timed Systems: benchmarks and statistics in this area (Fischer mutual exclusion, csma, etc.) are available <u>here;</u>
- Mutual exclusion, deadlock freedom and waiting time bounds for Fischer and Lynch-Shavit algorithms: see here for full analysis, statistics and benchmarks;
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Instructions

You can run MCMT via command line; for detailed explanations, please, have a look at the <u>User Manual</u>. MCMT must be linked to the SMT solver <u>Yices</u>.

Main Publications

• Theoretical framework.

S. Ghilardi, E. Nicolini, S. Ranise, and D. Zucchelli. <u>Towards SMT</u> <u>Model-Checking of Array-based Systems</u> In Proc. of IJCAR'08 (<u>extended version</u>).

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• Stopping failures / Approximated Models.

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Abstraction and acceleration features.
F. Alberti, R. Bruttomesso, S, Ghilardi, S. Ranise, N. Sharygina Lazy



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Technique and tool designed by Silvio Ranise and

myself

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. . .

THANKS FOR ATTENTION !

GIORNATA DELLA FIMA

FEDERAZIONE ITALIANA DI MATEMATICA APPLICATA



Venerdi 27 Ottobre 2017 ore 10.00 AULA MARCONI – CNR, PIAZZALE ALDO MORO, 7 ROMA

Internal resonances in aeroelastic oscillations

Giuseppe Piccardo



Dept of Civil, Chemical and Environmental Engineering DICCA – University of Genoa (Italy)





In cooperation with Prof. Angelo Luongo, University of L'Aquila

Outline

Only a short introduction on this wide subject, related to my personal experience on the topic.

Problems that may be of interest not only for engineer but also (or more) for applied mathematics (interdisciplinary topic)



Internal resonance



Fluid-structure interaction problems (wind)



Examples of applications Linear 1:1 Nonlinear 1:2



Some prospects

Internal resonances in aeroelastic oscillations

Example of nonlinear discrete dynamical system

$$\begin{split} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \mathbf{n}_{2m}(\mathbf{q},\mathbf{q}) + \mathbf{n}_{3m}(\mathbf{q},\mathbf{q},\mathbf{q}) + \mathbf{n}_{2a}(\dot{\mathbf{q}},\dot{\mathbf{q}};\boldsymbol{\mu}) + \mathbf{n}_{3a}(\dot{\mathbf{q}},\dot{\mathbf{q}},\dot{\mathbf{q}};\boldsymbol{\mu}) = \mathbf{f}(t) \\ & \text{nl mechanical terms} \\ \end{split}$$

Internal Resonances

Nayfeh & Mook – Nonlinear Oscillations (1979) Chapter 6 – Systems having Finite Degrees of Freedom

"In contrast with a single dof, which has only a single linear natural frequency and a single mode of motion, an *n*-dof system has *n* linear natural frequencies and *n* corresponding modes. Let us denote these frequencies by $\omega_1, \omega_2, ..., \omega_n$ and assume that all of them are real and different from zero. An important case occurs whenever two or more are commensurable or nearly commensurable. Examples of near-commensurability are"

 $\omega_2 \approx 2 \,\omega_1, \ \omega_2 \approx 3 \,\omega_1, \ \omega_3 \approx \omega_2 \pm \omega_1,$

 $\omega_3 \approx 2\omega_2 \pm \omega_1, \quad \omega_4 \approx \omega_3 \pm \omega_2 \pm \omega_1$

Nayfeh & Mook – Nonlinear Oscillations (1979) Chapter 6 – Systems having Finite Degrees of Freedom

Depending on the order of the nonlinearity in the system, these commensurable relationships of frequencies can cause the corresponding modes to be strongly coupled, and an internal resonance is said to exist. For example, if the system has quadratic nonlinearities, then to first order an internal resonance can exist if $\omega_m \approx 2\omega_k$ or $\omega_q \approx \omega_p \pm \omega_m$. For a system with cubic nonlinearities, to first order an internal resonance can exist if $\omega_m \approx 3\omega_k$ or $\omega_q \approx 2\omega_p \pm \omega_m$ or $\omega_q \approx \omega_p \neq \omega_m \neq \omega_k$. When an internal resonance exists in a free system, energy imparted initially to one of the modes involved in the internal resonance will be continuously exchanged among all the modes involved in that internal resonance. If damping is present in the system, then the energy will be continuously reduced as it is being exchanged. (positive)

Can internal resonance produce instability of slender structures?

Input energy is required

e.g., Fluid-Structure Interaction Problems

Blevins (2001)

Feedback

FORCE ON STRUCTURE

(magnitude, coherence, time lag)

FORCE DYNAMICS

(velocity, density, e.g. turbulence and vortices in fluid) STRUCTURAL DYNAMICS

(natural frequency, mass, damping, vibration amplitude)

LOAD/STRUCTURE BOUNDARY MOTION

(oscillation separation, local surface velocity, e.g. instantaneous angle of attack for wind)

G. Piccardo

Internal resonances in aeroelastic oscillations

Proposal of "classification" of the aeroelastic phenomena (by Matsumoto, 1996)



In general the physics of this phenomena is very complex, they are the subject of accurate <u>experimental</u> investigations

Internal resonances in aeroelastic oscillations



The simplest possible model (1dof galloping model)

Flexibly supported bluff body (typically non-circular section)

We define galloping as a velocitydependent, damping-controlled instability, giving rise to transversal oscillations (Parkinson & Smith 1964)

The **quasi-steady theory** assumes that the forces acting on a moving cylinder can be predicted adopting the expression pertinent to a fixed cylinder in which the **asymptotic flow velocity is substituted by the flow-cylinder relative velocity**

Quasi-steady modeling of aeroelastic actions

From a physical point of view this means that <u>the forces are determined only</u> by the instantaneous geometry and the instantaneous velocity field of the <u>flow around the cylinder</u>, and that any memory effect is negligible

The name Galloping seems quite appropriate because of the visual impression given when it occurs in transmission lines: typically a low-frequency highamplitude oscillation, reminiscent of a galloping horse





Internal resonances in aeroelastic oscillations

Quasi-steady modeling of aeroelastic actions

This hypothesis may be satisfied if the characteristic times of the velocity fluctuations in the wake of the cylinder are much smaller than the characteristic times of the cylinder's oscillating motion:



 f_{vs} being the vortex-shedding frequency, St the Strouhal number, f_c the cylinder's oscillation frequency.

Since the *St* assumes values approximately ranging between 0.1-0.2 for the most common cross-section shapes, this requirement is often met at high reduced velocity U_r :

$$U_r = \frac{U}{f_c b} >> \frac{1}{St} \Longrightarrow U_r > 20$$

This limit is therefore empirical and is disputed by some scientists who consider it should be much higher. Furthermore, <u>the use of quasi-steady approach is questionable</u> <u>when reduced velocities are not sufficiently high</u> because steady forces induced by instantaneous changes in angle of attack of the mean wind are influenced (or obscured) by unsteady vortex shedding forces acting at similar frequencies.

G. Piccardo

Internal resonances in aeroelastic oscillations

Quasi-steady modeling of aeroelastic actions



Wind Tunnel Static Test (DICCA WT, Genoa University)



- End-plates at the extremities of the model to ensure 2D flow conditions
- St estimated by fitting the PSD function of 1.5 the lift force through a Gaussian function in 1 the neighborhood of the vs peak 0.5
- aerodynamic coefficients derived both from ⁰ force measurements (force balances -0.5 applied at the end of the model) and from -1 pressure measurements (pressure taps applied to the centerline of the model)





G. Piccardo

Internal resonances in aeroelastic oscillations

VIV

Vortex-induced vibrations constitutes a key element in the cross-wind response of structure



Fundamental non-dimensional parameters

 $St = \frac{n_s b}{U}$ <u>Strouhal number</u>

function of Reynolds number and roughness for circular cylinders

<u>Scruton number</u> $Sc = \frac{4\pi m\xi}{\rho b^2}$

Resonance condition in cross-wind direction, i.e. $n_s = n_v$

The **critical velocity** referred to the i^{th} mode can be evaluated as

 $U_{cr,i} = \frac{n_{y,i}b}{St}$

<u>Cables, chimneys, slender structures</u>: low critical velocities, more modes involved, fatigue problem

All **VIV models** in the literature are <u>empirical in nature</u>, trying to capture the essential features of the phenomenon, and usually involve only one structural degree-of-freedom (the crosswind one). They can be classified into <u>three main categories</u> (see, e.g., Paidoussis, Price, de Langre):

- **forced-system models**, in which the force is independent of the structural motion, and therefore only depends on time (e.g., harmonic model)
- **fluid-elastic system models**, where the force depends on the structural motion (i.e. velocity and displacement of the system; e.g., spectral model)
- coupled system models, in which the force depends on an additional variable related to the wake dynamics and a second equation related to the fluid oscillator is coupled with the structural motion equation ("wakeoscillator models").

<u>Fluid-elastic system models</u> have been widely adopted in the literature because of their simplicity combined with their <u>ability to reproduce the essential features</u> of lock-in vibrations (i.e. the self-sustained flow instability and its self-limitation in amplitude)

Fluid-elastic system models

From a mathematical viewpoint, the classic van der Pol (or, equivalently, Rayleigh) equation is able to capture these features; alternatively, the famous Ginzburg-Landau equation can be used (see Schewe, JFS 2013), whose equivalence in the description of the physical phenomenon can be roughly proved through the multiple scale method applied to Rayleigh or van der Pol oscillators

$$\frac{d^2 x}{dt^2} - \mu (1 - x^2) \frac{dx}{dt} + x = 0$$

wind engineering
$$\xi_a = -\xi_s \frac{K_{a0}(k)}{Sc} \left[1 - \varepsilon(k) \frac{x^2}{D^2} \right]$$
$$= \xi_s \left[1 - \frac{K_{a0}(k)}{Sc} \left(1 - \frac{\sigma_x^2}{(\lambda(k)D)^2} \right) \right]$$



Phase portrait of the unforced vdP oscillator, showing a limit cycle and the direction field

Vickery-Basu approach (1983) Equivalent linearization of nl damping (nonlinearity now appears in statistical form)

the STD of response can be estimated based on classic random vibration theory of SDOF

 ξ_{eq}

Coupled-system models

<u>Modelling of coupled cross-flow/in-line vortex-induced vibrations using double Duffing</u> <u>and van der Pol oscillators</u> by Srinil & Zanganeh 2012 (Ocean Engineering)



The cubic nonlinearities capture the geometrical coupling of cross-flow/in-line displacements excited by hydrodynamic lift/drag forces whereas the quadratic nonlinearities allow the wake– cylinder interactions. Some empirical coefficients are calibrated against published experimental results to establish a new generic analytical function accounting for the dependence of VIV on a physical mass and/or damping parameter.

Wind Tunnel Aeroelastic Free/Forced Test (DICCA WT, Genoa University)

- damping ratio in the range 0.038-0.15% using the eddy currents method by means of an electromagnet
- Model displacements have been measured through laser sensors sampled at 2 kHz.





- experimental rig to test prisms moving in crossflow direction with harmonic motion
- ➡ pressure measurements during motion at different wind speeds V = 5 20 m/s ($V_r = 5.9 200$, amplitudes Y = 10%, 20% and 30%, motion frequencies 0.7 17.5 Hz
- motion-excited forces in terms of amplitude and phase

Internal resonances in aeroelastic oscillations

Linear instability mechanisms for coupled translational galloping (Luongo & Piccardo, JSV 2005)

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{D}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = 0,$$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} \omega^2 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{q}(t) = \begin{cases} q_1(t) \\ q_2(t) \end{cases}$$

aeroelastic effects

internel recence



system, valid for any frequency ratio ω

 $\mathbf{D} = \epsilon \tilde{\mathbf{D}}$

. A perturbation parameter $\varepsilon << 1$ is introduced through a suitable ordering of the coefficients. Namely, it is assumed that the coefficients of the total damping matrix **D** are small of order ε

Two different cases, according to the frequency ratio ω :

(a) non-resonant solution, $|\omega - 1| > O(\varepsilon)$

(b) resonant and quasi-resonant solution, $|\omega - 1| = O(\varepsilon)$

Case (a)
$$\varepsilon^0 : [\mathbf{K} + \lambda_0^2 \mathbf{I}] \mathbf{w}_0 = 0,$$
 Perturbation equations
 $\varepsilon : [\mathbf{K} + \lambda_0^2 \mathbf{I}] \mathbf{w}_1 = -2\lambda_0\lambda_1 \mathbf{w}_0 - \lambda_0 \mathbf{D} \mathbf{w}_0$

Order- ε eq. is a singular non-homogenous problem. In order that it admits solution, the known-term must belong to the range of the operator (solvability, or compatibility, condition), i.e. it must be orthogonal to the null-space of the adjoint operator. Since the operator is self-adjoint, orthogonality to $\mathbf{w}_0^{(k)}$ must be enforced. From this condition, the first-order eigenvalue sensitivity $\lambda_1 = \lambda_1^{(k)}$ is drawn. Then, the first-order eigenvector sensitivity $\mathbf{w}_1 = \mathbf{w}_1^{(k)}$ is evaluated and the arbitrary constant can be removed by a suitable normalization condition

$$\lambda^{(1,2)} = \pm i\omega - \frac{1}{2}d_{11}, \quad \mathbf{w}^{(1,2)} = \left(1, \pm \frac{i\omega d_{21}}{1-\omega^2}\right)^{\mathrm{T}},$$

resonant solution
$$\lambda^{(3,4)} = \pm i - \frac{1}{2}d_{22}, \quad \mathbf{w}^{(3,4)} = \left(\pm \frac{id_{12}}{\omega^2 - 1}, 1\right)^{\mathrm{T}}$$

Internal resonances in aeroelastic oscillations

Case (b) resonant and quasi-resonant solution

$$\omega = 1 + \varepsilon \sigma, \ \sigma = O(1)$$
detuning
$$\mathbf{K} = \mathbf{K}_0 + \varepsilon \mathbf{K}_1 + O(\varepsilon^2), \quad \mathbf{D} = \mathbf{D}_0 + O(\varepsilon),$$

$$\mathbf{K}_0 = \mathbf{I}, \quad \mathbf{K}_1 = \begin{bmatrix} 2\sigma & 0\\ 0 & 0 \end{bmatrix}, \quad \mathbf{D}_0 = 2\xi_s \mathbf{I} + \mu \mathbf{C}_a$$

Perturbation equations

$$\varepsilon^{0} : [\mathbf{I} + \lambda_{0}^{2}\mathbf{I}]\mathbf{w}_{0} = 0,$$

$$\varepsilon : [\mathbf{I} + \lambda_{0}^{2}\mathbf{I}]\mathbf{w}_{1} = -2\lambda_{0}\lambda_{1}\mathbf{w}_{0} - \mathbf{K}_{1}\mathbf{w}_{0} - \lambda_{0}\mathbf{D}_{0}\mathbf{w}_{0}$$

Since $\lambda_0^{(1,2)} = i$, $\lambda_0^{(3,4)} = -i$ are eigenvalues of double multiplicity for order-1 eq., the associated eigenvectors are indeterminate, namely:

 $\mathbf{w}_{0} = \sum_{i=1}^{2} \alpha_{i} \mathbf{u}_{i}, \quad \mathbf{u}_{1} = \begin{cases} 1\\0 \end{cases}, \quad \mathbf{u}_{2} = \begin{cases} 0\\1 \end{cases}$ In order that order- ε eq. admits solution, its right-hand term must be orthogonal to each of \mathbf{u}_{i} , i.e.: $\sum_{i=1}^{2} \left(2\lambda_{0}\lambda_{1}\mathbf{u}_{j}^{\mathrm{T}}\mathbf{u}_{i} + \mathbf{u}_{j}^{\mathrm{T}}\mathbf{K}_{1}\mathbf{u}_{i} + \lambda_{0}\mathbf{u}_{j}^{\mathrm{T}}\mathbf{D}_{0}\mathbf{u}_{i} \right) \alpha_{i} = 0, \quad j = 1, 2.$

It represents a new eigenvalue problem. By setting its determinant to zero, two generally different corrections of each 1st order-eigenvalue are obtained, i.e. the ε -order perturbation destroy the degeneracy.

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Internal resonances in aeroelastic oscillations

Case (b) resonant and quasi-resonant solution

Finally, using the normalization and reabsorbing the ordering parameter ε , the following approximate eigensolutions are obtained:

$$\lambda^{(1,2)} = i - \xi_s + \frac{1}{4} \left(-\mu \operatorname{tr} \mathbf{C}_a + 2i\sigma \pm \sqrt{\mu^2 (\operatorname{tr}^2 \mathbf{C}_a - 4 \det \mathbf{C}_a) - 4i\sigma (d_{11}^0 - d_{22}^0) - 4\sigma^2} \right),$$

$$\lambda^{(3,4)} = -i - \xi_s + \frac{1}{4} \left(-\mu \operatorname{tr} \mathbf{C}_a - 2i\sigma \pm \sqrt{\mu^2 (\operatorname{tr}^2 \mathbf{C}_a - 4 \det \mathbf{C}_a) + 4i\sigma (d_{11}^0 - d_{22}^0) - 4\sigma^2} \right)$$

$$\mathbf{w}^{(1,2)} = \left(-\frac{\mathrm{i}d_{12}^0}{2\mathrm{i}(\lambda^{(1,2)} - \mathrm{i}) + 2\sigma + \mathrm{i}d_{11}^0}, -1 \right)^T$$
$$\mathbf{w}^{(3,4)} = \left(\frac{\mathrm{i}d_{12}^0}{-2\mathrm{i}(\lambda^{(3,4)} + \mathrm{i}) + 2\sigma - \mathrm{i}d_{11}^0}, -1 \right)^T$$

Eigenvalue paths for increasing wind velocities in the plane of the <u>aerodynamic damping</u> <u>matrix invariants</u>; the parabola has equation

$$\mathrm{tr}^2\mathbf{C}_a - 4\mathrm{det}\,\mathbf{C}_a = 0$$

the use of the aerodynamic damping matrix C_a makes explicit the role of the dimensionless wind velocity μ



Internal resonances in aeroelastic oscillations

resonant solution

the lowest critical wind velocity as a function of the C_a -invariants

$$\mu_{\rm cr}^0 = \begin{cases} \frac{4\xi_s}{-\operatorname{tr} \mathbf{C}_a + \sqrt{\operatorname{tr} \mathbf{C}_a^2 - 4\operatorname{det} \mathbf{C}_a}} & \text{if det } \mathbf{C}_a < 0 \text{ or } \operatorname{tr}^2 \mathbf{C}_a - 4\operatorname{det} \mathbf{C}_a > 0, \text{ tr } \mathbf{C}_a < 0, \\ -\frac{4\xi_s}{\operatorname{tr} \mathbf{C}_a} & \text{if } \operatorname{tr} \mathbf{C}_a^2 - 4\operatorname{det} \mathbf{C}_a \leqslant 0, \text{ tr } \mathbf{C}_a < 0, \end{cases}$$

Critical wind velocity in the resonant case compared with the Den Hartog value



Internal resonances in aeroelastic oscillations

quasi-resonant solution

summarizes all the possible critical conditions exhibited by the quasi-resonant system

 trC_a Eigenvalue paths for increasing wind velocities in the plane of the (A) aerodynamic damping matrix invariants (D) C parabola equation $\operatorname{tr}^{2}\mathbf{C}_{a} - (4 + \sigma^{2}/\xi_{s}^{2}) \det \mathbf{C}_{a} = 0$ $detC_a$ (B) (D) (C)**Effect of internal resonance** μ_{cr} μ_{cr} μ_{cr} μ_{cr} (C),(D) μ_{DH} (D) (A) **(B)** (D) μ_{DH} σ σ μ_{DH} σ 0 σ 0 $c_{22} < 0$ (a) (b) $c_{22} > 0$

Internal resonances in aeroelastic oscillations

A <u>Continuous</u> Approach to the Aeroelastic Stability of Suspended Cables in 1:2 Internal Resonance Luongo & Piccardo, JVC 2008

first symmetric in-plane frequency is double the first symmetric out-of-plane frequency (first cross-over point)



Internal resonances in aeroelastic oscillations



Perturbation solution directly on the nonlinear continuum equations, avoid an 'a priori' discretization of the problem

determination of the non-trivial path related to the static equilibrium

writing the equations of motion around this equilibrium position

evaluation of the influence of the equilibrium path on aerodynamic forces

Equations of motion around $\overline{\mathscr{C}}$ (reference cfg)

- small sag-span ratio (parabolic profile, curvature \cong constant, stress \cong constant)
- Iongitudinal inertia neglected (static condensation)

$$u_{1}'' - \Omega^{2} \ddot{u}_{1} = \alpha u_{1}'' \left[\beta \int_{0}^{1} u_{2} ds - \frac{1}{2} \int_{0}^{1} (u_{1}'^{2} + u_{2}'^{2}) ds \right] - b_{1} (\dot{u}_{1}, \dot{u}_{2})$$

$$u_{2}'' - \alpha \beta^{2} \int_{0}^{1} u_{2} ds - \Omega^{2} \ddot{u}_{2} = \alpha u_{2}'' \left[\beta \int_{0}^{1} u_{2} ds - \frac{1}{2} \int_{0}^{1} (u_{1}'^{2} + u_{2}'^{2}) ds \right] - \frac{\alpha \beta}{2} \int_{0}^{1} (u_{1}'^{2} + u_{2}'^{2}) ds - b_{2} (\dot{u}_{1}, \dot{u}_{2})$$



rotation ϕ

Internal resonances in aeroelastic oscillations

The equations of motion are solved through the <u>Multiple Scale Method</u>. It is well known that this method furnishes the bifurcation equations that rule the time evolution of amplitude and phase related to active modes (e.g., Luongo et al., 2003). When it is directly applied to pde, <u>it automatically takes into account the contribution of</u> <u>passive modes</u>, through the origin of "**secondary modes**", that are responsible for the spatial alteration of oscillation shapes, induced by the presence of nonlinearities

- Asymptotic expansion
- Independent time scales
- Resonance condition

$$\underline{\mathbf{u}} = \varepsilon \mathbf{u}_1 + \varepsilon^2 \mathbf{u}_2 + \varepsilon^3 \mathbf{u}_3, \quad \underline{\mathbf{u}} = (u_1, u_2)$$
$$t_0 = t, \quad t_1 = \varepsilon t, \quad t_2 = \varepsilon^2 t, \quad \dots$$
$$\omega_2 = 2\omega_1 + \varepsilon\sigma, \quad \sigma = O(1)$$



1st mode (symmetric in-plane) Internal resonances in aeroelastic oscillations
<u>Perturbation equations</u>

 \checkmark order ϵ

 $\mathbf{\underline{L}} - \mathbf{\underline{d}}_0^2 \mathbf{\underline{u}}_1 = \mathbf{\underline{0}}$ $\mathbf{\underline{B}} \mathbf{\underline{u}}_1 = \mathbf{\underline{0}}$

$$\mathbf{A}_{k} = \partial / \partial$$

 \checkmark order ε^3

$$\begin{cases} (\underline{\mathbf{L}} - \mathbf{d}_0^2)\underline{\mathbf{u}}_3 = 2\mathbf{d}_0\mathbf{d}_1\underline{\mathbf{u}}_2 + \mathbf{d}_1^2\underline{\mathbf{u}}_1 + 2\mathbf{d}_0\mathbf{d}_2\underline{\mathbf{u}}_1 + 2N_2(\underline{\mathbf{u}}_1, \underline{\mathbf{u}}_2) + N_3(\underline{\mathbf{u}}_1, \underline{\mathbf{u}}_1, \underline{\mathbf{u}}_1, \underline{\mathbf{u}}_1) \\ \underline{\mathbf{B}}\underline{\mathbf{u}}_3 = \underline{\mathbf{0}} \end{cases}$$

Solution of order ε

$$\underline{\mathbf{u}}_{1} = \begin{pmatrix} A_{1}(t_{1}, t_{2}) \,\phi_{1}(s) \,e^{\mathrm{i}\,\omega_{1}t_{0}} \\ A_{2}(t_{1}, t_{2}) \,\phi_{2}(s) \,e^{\mathrm{i}\,\omega_{2}t_{0}} \end{pmatrix}$$

eigenfunctions of the Hamiltonian system

Internal resonances in aeroelastic oscillations

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- Solution of order ε²
 - Solvability condition

$$d_1 \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \underline{\mathbf{f}}_1 (A_1, A_2)$$

Amplitude modulation on t_1

$$\int_{0} \int_{0} \left[2d_0 d_1 \underline{\mathbf{u}}_1 + N_2(\underline{\mathbf{u}}_1, \underline{\mathbf{u}}_1) \right] \phi_k(s) e^{i\omega_k t_0} \, \mathrm{d}s \, \mathrm{d}t_0 = 0 \quad (k = 1, 2)$$

"secondary modes" describing the alteration of the spatial shape of the response, each of which is associated with a harmonic that takes part in the motion (base, sum and difference)

 $2\pi/\omega_k$ 1

$$\mathbf{\underline{u}}_{2} = \begin{pmatrix} 0 \\ i A_{1} \psi_{1}(s) \end{pmatrix} e^{i \omega_{1} t_{0}} + \begin{pmatrix} i A_{2} \psi_{2}(s) \\ A_{1}^{2} \psi_{11}(s) e^{-i \sigma t_{1}} \end{pmatrix} e^{i \omega_{2} t_{0}} + \\ + \begin{pmatrix} A_{1} A_{2} \psi_{12}(s) \\ 0 \end{pmatrix} e^{i (\omega_{1} + \omega_{2}) t_{0}} + \begin{pmatrix} 0 \\ A_{2}^{2} \psi_{22}(s) \end{pmatrix} e^{2i \omega_{2} t_{0}} + \\ + \begin{pmatrix} 0 \\ A_{1} \overline{A}_{1} \psi_{1\overline{1}}(s) + A_{2} \overline{A}_{2} \psi_{2\overline{2}}(s) \end{pmatrix} + c.c.$$

The shape functions $\psi(s)$ are solutions of ordinary differential (linear) problems and contain the contribution of <u>PASSIVE MODES</u>

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Internal resonances in aeroelastic oscillations

<u>Solution of order ε³</u>

✓ solvability $d_2 \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \underline{\mathbf{f}}_2(A_1, A_2)$ Amplitude modulation on t_2 Bifurcation equations Amplitude $D\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \varepsilon d_1 \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} + \varepsilon^2 d_2 \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$ ✓ reconstitution modulation on t polar representation $A_k = \frac{1}{2} a_k e^{i \vartheta_k}$ k = 1, 2 $\delta = \vartheta_2 - 2\vartheta_1 + \sigma t$ autonomous form of AME (real bifurcation eqs) phase difference $\dot{a}_{1} = a_{1} \left[\frac{c_{11}}{2\Omega^{2}} + \frac{1}{2}a_{2}(p_{21}\cos\delta - p_{22}^{*}\sin\delta) + \frac{1}{4}a_{2}^{2}p_{31} + \frac{1}{4}a_{1}^{2}p_{41} \right]$ $\dot{a}_{2} = a_{2} \frac{c_{22}}{2\Omega^{2}} + \frac{1}{2} a_{1}^{2} (p_{61} \cos \delta + p_{62}^{*} \sin \delta) + \frac{1}{4} a_{1}^{2} a_{2} p_{71} + \frac{1}{4} a_{2}^{3} p_{81}$ $a_{1}a_{2}\dot{\delta} = a_{1}\left[a_{2}(p_{5}-2p_{1}+\sigma)-a_{2}^{2}(p_{21}\sin\delta+p_{22}^{*}\cos\delta)+\frac{1}{2}a_{1}^{2}(-p_{61}\sin\delta+p_{62}^{*}\cos\delta)+\frac{1}{4}a_{2}^{3}(p_{82}-2p_{32})+\frac{1}{4}a_{1}^{2}a_{2}(p_{72}-2p_{42})\right]$

Continuation and Bifurcation Software for Ordinary Differential Equation, AUTO

Internal resonances in aeroelastic oscillations



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Internal resonances in aeroelastic oscillations



-0.008-0.012-0.012 -0.008 -0.004 0 0.004 0.008 0.012 u_1

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Internal resonances in aeroelastic oscillations



Internal resonances in aeroelastic oscillations

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Possible developments – Nonlinear Galloping

$$\underline{\mathbf{u}}_{1} = \begin{pmatrix} A_{1}(t_{1},t_{2}) \phi_{1}(s) e^{i\omega_{1}t} + A_{3}(t_{1},t_{2}) \phi_{3}(s) e^{2i\omega_{1}t} \\ A_{2}(t_{1},t_{2}) \phi_{2}(s) e^{i\omega_{2}t} + A_{4}(t_{1},t_{2}) \phi_{4}(s) e^{2i\omega_{1}t} \end{pmatrix}$$



Perturbation equations of order ε^2 and ε^3 move from a number of 2 to a number of 4, in the new unknowns v and w (symmetric and skew-symmetric parts of displacements)



It was determined the new autonomous form of AME : 7 nonlinear algebraic equations in the unknowns $(a_1, a_2, a_3, a_4) \in (\gamma_1, \gamma_2, \gamma_3)$

It has been found a path purely skew-symmetric, which presents a bifurcation point very close to that of symmetric paths

multiple bifurcation seems to actually exist

Some problems under investigation

Dry galloping in inclined cables

(Piccardo, Zulli, Luongo, Eurodyn 2017)

(inclined cables/hybrid modes, influence of internal resonances)

A warpable equivalent beam model for the analysis of tower buildings (Luongo, Piccardo, AIMETA 2017)

Wind- and Vortex-induced Vibrations in slender/supertall buildings (Pagnini, Piccardo, JFS 2017; in preparation)

Quasi-steady vs unsteady forces

Thunderstorm effects on structures/cables

ERC Advanced Grant (AdG) 2016 Prof. Solari – Univ. Genova "Detection, simulation, modelling and loading of thunderstorm outflows to design wind-safer and cost-efficient structures"





T₁*≅*15 s

Internal resonances in aeroelastic oscillations

G. Piccardo

Some problems under investigation



Internal resonances in aeroelastic oscillations



The aim of GADeS is to promote the scientific interaction between researchers operating in the fields of Solid and Structural Dynamics, Machine Dynamics and Dynamical Systems, as well as the opportunity for improving cooperation with researchers working on Identification and Control problems, **independently from the scientific sector**, thus in coherence with the AIMETA's statutory spirit.

Indeed, the **integration of researchers, who are studying particular and/or complementary problems**, appears very important since it can enlarge the range of interests of the Group. Therefore, the AIMETA GADeS Group is specifically aimed at sharing knowledge from different sectors, through the organization of workshop, mini-symposia, special sessions in AIMETA's conferences and, last but not least, the coordinated participation to Italian and European research project.

GADeS – 2017 activities

Stability and Bifurcation of Dynamical Systems: Theoretical Aspects and Applications

July 3-7, 2017, Savona, Italy

Main speakers:

Walter D'Ambrogio (Univ. L'Aquila): 'Numerical analysis of friction-induced instabilities'
Claudio Giorgi (Univ. Brescia): 'Theory of dynamical system'
Angelo Luongo (M&MoCS, Univ. L'Aquila): 'Bifurcation analysis and perturbation methods'
Michele Ciavarella (Univ. Bari): 'Thermoelastic contact instabilities'
Stefania Gatti (Univ. Modena e Reggio Emilia) 'Global study of the asymptotic behavior'
Daniele Zulli (M&MoCS, Univ. L'Aquila): 'Numerical methods in bifurcation analysis'

AIMETA 2017, Salerno (Sept 2017 – GADeS Mini-Symposium)

dedicated to the memory of Prof. Ali H. Nayfeh

45 presentations

9 sessions

- \cong 1 Applied Mathematics
- \cong 3 Applied (Machine) Mechanics
- \cong 5 Structural Mechanics

BIG DATA and BUSINESS ANAlytics, A Critical Overview

Giornata Della FIMA, 27 ottobre 2017, CnR Roma

Emanuele Borgonovo Bocconi University and AMASES Director Bachelor in Economics, Management and Computer Science Co-Editor in Chief, European Journal of Operational Research



Digitalization

We are at the verge of the data driven economy Globalization and increased connectivity make available to citizens, to corporations and scientists an amount of data unthinkable a few years ago

We register a large increase in the use of computers in everyday life, with new societal phenomena, entrepreneurial opportunities and new professions



Prevasiveness





Parliament

- July 2 2014
- Several players to create added value from the availability of big data
- EC document on cloud computing: big data and correlated services will reach 16,9 billion USD value in 2015, with an average growth rate of 40%, seven times higher the rate of growth of technology market
- In the UK number of big data specialists working in big firms will increase by 240% (Source: SAS report)





"McKinsey predicts that companies will struggle to find Big Data talent due to a shortage in well-trained people."

EU Document



New Professions and New Skills

D.J. Patil nominated in February 2015

first White House chief data officer

Data Science Team for the Italian Digital Agenda





New Programs Nationally and Internationally

Bachelor Level: a few programs (Bocconi, Milan; Bologna) MSc: 11 programs (source: MastersPortal.eu): Bologna, Genova, Milan (Bocconi, Bicocca, Cattolica, Statale), Roma (Luiss, Tor Vergata, Sapienza), Padova, Torino



Pillars



What is **Big Data?**

Big Data is high-volume, high-velocity and high-variety information assets that demand cost-effective, innovative forms of information processing for enhanced insight and decision making (Source: Gartner)

- the technical skills to gather and manage data
- the ability to **interpret** data through the lens of economic models, thus enabling them to provide credible support to company decision making.



Computer Science





Algorithms

Click Here





Page Rank





Machine Learning: Algorithms

The dataset is divided into training and testing

The data are not prepared or labelled, the algorithm learns by similarity



Types of Algorithms... (if a list is possible...)





THE DANTZIG SELECTOR: STATISTICAL ESTIMATION WHEN pIS MUCH LARGER THAN n^1

BY EMMANUEL CANDES² AND TERENCE TAO³

California Institute of Technology and University of California, Los Angeles

$$\min_{\tilde{\beta} \in \mathbf{R}^p} \|\tilde{\beta}\|_{\ell_1} \quad \text{subject to} \quad \|X^* r\|_{\ell_\infty} \le (1+t^{-1})\sqrt{2\log p} \cdot \sigma,$$

Again, the program (DS) is convex, and can easily be recast as a linear program (LP),

(1.9)
$$\min \sum_{i} u_{i} \text{ subject to } -u \leq \tilde{\beta} \leq u \text{ and}$$
$$-\lambda_{p} \sigma \mathbf{1} \leq X^{*} (y - X \tilde{\beta}) \leq \lambda_{p} \sigma \mathbf{1},$$

B Università Bocconi _{MILANO}

where the optimization variables are $u, \tilde{\beta} \in \mathbb{R}^p$, and **1** is a *p*-dimensional vector of ones. Hence, our estimation procedure is computationally tractable; see Section 4.4

Some most Recent Examples

A. Lodi, Y. Banjo, E. Frejinger (Montreal) Using AI to help solve complex optimization problems Helps in Mixed Integer-Linear programs Helps in Integer programs improving branch and bound

Borgonovo, E., Lu, X. Rosasco A., Rudi A. Fast kriging

Select your best camera Multicriteria problem Bayesian Approach: set a prior over a class of uncertain parameters of the algorithm Computer Implementation



Challenges



Large Donations



Business Impact: Industry 4.0





Social Impact

- New jobs for young generations?
- What about the intermediate generation?
- Need for additional education?
- How does society perceives AI?



Algorithms (models) can fail



Epistemologically, what are we doing?





Scientific Challenges



GRAZIE Per LA VOSTRA Attenzione!




A nonlinear model for marble sulphating including surface rugosity: analytical and numerical results

<u>Elena Bonetti</u> University of Milan

joint research with

<u>Cecilia Cavaterra</u> (University of Milan) <u>Maurizio Grasselli</u> (Politecnico of Milan) <u>Francesco Freddi</u> (University of Parma) <u>Roberto Natalini</u> (IAC-CNR, Rome)

Workshop FIMA - Roma, 27 ottobre 2017

Conservation and restoration of historical buildings



Mathematical models

Accademia Nazionale dei Lincei SDSM - Stone Durability for a Sustainable Maintenance

Forecasting:

- stone durability (damages)
- impact of restoration interventions

Expensive, wasteful, complex

Statistical methods (Lipfert's formula), but average unsuitable values

Model:

- analytical and computational tools
- on the basis of physical evidence and data

To suggest how to schedule improved strategies of conservation and consolidation interventions on monumental stones

Focus more on data

- Physico-chemical data about stones: porosity, permeability, shape and size of crystals, chemical composition, ...
- Environmental data: temperature, humidity, pollution (SOx, NOx, Co2), rain, wind...
- Surface damage: rugosity, porosity, ...
- 3D geometry and shape
- Mechanical properties, ...





The importance of environment: deposition of pollution



Deterioration of marble: the SO2 effect



The mechanism: SO2 + CaCO3





Sulphation crusts



The model





A simple model (Natalini, Freddi et al.): c CaCO3 density, s SO2 porous density

Hydrodynamical models: accurate numerical approximation by finite elements or finite differences methods



Understanding of physical process, adaptation of the model to more complex situations (including damage factors), affective calibration against experimental data

The increasing of the crust



Crust increases with an order of \sqrt{t}

Consequence: removing gypsum increases the amount of lost marble



A Project: II Vittoriano (Rome) 2009-2011



-0.5

Thickness (cm)

Marble

1.5

к 10⁻⁵

0.5

0.4

0.2

-1.5



Funded by Ministero dei Beni Culturali Main Sponsor: Direzione generale dei Beni culturali del Lazio Project Leader: E. Giani (ISRC-MIBAC) Participants:

ISCR - MIBAC: C. Cacace, A. Giovagnoli, L. Gordini CISTeC - "Sapienza" Univ. Roma : M.L. Santarelli, P. Bracciale IAC - CNR: R. Natalini, F. Clarelli, B. De Filippo.

Inside the material (Natalini & Freddi)







Surface damage (rugosity) influencing the phenomenon



The equation for the rugosity evolution



Surface "damage" equation by use of a phase transition approach The evolution PDE system

Initial conditions

$$s(0) = s_0, \quad c(0) = c_0, \quad r(0) = r_0$$

A possible constraint to ensure r uniformly bounded and non-negative

$$\partial W(r) = \partial I_{[0,r_l]}(r) = \begin{cases} 0 \text{ if } r \in (0,r_l) \\ [0,+\infty) \text{ if } r = r_l \\ (-\infty,0] \text{ if } r = 0 \end{cases}$$

The choice of the potentials

 $\begin{array}{ll} \partial W(r)(\mathrm{e.g.}~\partial I_{[0,r_l]}) & \text{maximal montone operator (internal constraint)} \\ G & \text{depends on s and c through the porosity function} \\ \phi(c) & \text{the porosity function, it is assumed} \ \phi(c) = A + Bc \\ \lambda > 0 & \text{reaction coefficient} \\ \nu(\cdot) \geq 0 & \text{the permeability coefficient (increasing with r)} \\ \Psi'(r) & \text{possibly non-monotone dynamics on r} \end{array}$

The assumptions

 $W: [0, +\infty) \rightarrow [0, +\infty]$ is convex, and l.s.c., W(0) = 0 $\Psi \in W^{2,\infty}(\mathbb{R}), \quad G \in W^{1,\infty}(\mathbb{R}^3), \quad F \in L^2(0,T;L^2(\Gamma))$ $\bar{s} \in H^{1/2}(\Gamma), \quad \bar{s} > 0$ a.e. in Γ $\nu \in W^{1,\infty}(\mathbb{R}), \quad \nu \ge 0 \text{ a.e. in } \mathbb{R}$ $c_0 \in H^2(\Omega), \quad 0 \le c_0(x) \le C_0 \quad \forall x \in \Omega,$ $s_0 \in H^2(\Omega), \quad s_0(x) \ge 0 \ \forall x \in \Omega,$ $r_0 \in L^2(\Gamma), \quad W(r_0) \in L^1(\Gamma), \quad r_0 \ge 0$ a.e. in Γ $A > 0, A + BC_0 > 0$ $B \leq \frac{1}{S_0}$ and $\bar{s}(x) \leq S_0$ for a.a. $x \in \Gamma$, with $S_0 > 0$ s.t. $s_0(x) \leq S_0 \quad \forall x \in \Omega$. $s \in H^{1}(0, T; L^{2}(\Omega)) \cap L^{\infty}(0, T; H^{1}(\Omega)) \cap L^{2}(0, T; H^{2}(\Omega)),$ $c \in H^{1}(0, T; H^{2}(\Omega)),$ $r \in H^{1}(0, T; L^{2}(\Gamma)),$ $s \geq 0, \quad 0 \leq c \leq C_{0}, \quad \text{a.e. in } \Omega \times (0, T),$

$$\begin{aligned} \partial_t(\phi(c)s) &-\operatorname{div} \ (\phi(c)\nabla s) = -\phi(c)sc, \text{ a.e. in } \Omega \times (0,T), \\ \phi(c)\partial_n s &= -\nu(r)(s-\bar{s}), \text{ a.e. on } \Gamma \times (0,T), \\ \partial_t c &= -\phi(c)cs, \text{ a.e. in } \Omega \times (0,T), \\ \partial_t r + \xi + \Psi'(r) + G(r,c,s) = F, \quad \xi \in \partial W(r), \quad \text{a.e. on } \Gamma \times (0,T), \\ s(0) &= s_0, \ c(0) = c_0, \text{ a.e. in } \Omega, \qquad r(0) = r_0, \text{ a.e. on } \Gamma. \end{aligned}$$

The (global in time) existence result [B., Cavaterra, Freddi, Grasselli, Natalini, '17]

Under suitable assumptions there exists a unique solution in (0,T) fulfilling

 $s \in H^{1}(0,T;L^{2}(\Omega)) \cap L^{\infty}(0,T;H^{1}(\Omega)) \cap L^{2}(0,T;H^{2}(\Omega)),$ $c \in H^{1}(0,T;H^{2}(\Omega)),$ $r \in H^{1}(0,T;L^{2}(\Gamma))$ $s \in W^{1,\infty}(0,T;L^{2}(\Omega)) \cap H^{1}(0,T;H^{1}(\Omega)) \cap L^{\infty}(0,T;H^{2}(\Omega)),$ $c \in W^{1,\infty}(0,T;H^{2}(\Omega)),$ $0 \leq s \leq S_{0}, \quad 0 \leq c \leq C_{0} \quad \text{a.e. in } \Omega \times (0,T),$ $\xi \in L^{2}(0,T;L^{2}(\Gamma)).$

1. Local in time result by a fixed point theorem (contraction argument): a Banach theorem for a suitable norm to ensure (at least for small time interval) that we can define an operator on

$$\mathcal{X}_{R,T} := \{ s \in L^2(0,T; H^2(\Omega)) : s \ge 0, \|s\|_{L^{\infty}([0,T]:L^2(\Omega)) \cap L^2(0,T; H^2(\Omega)) \le R} \}$$

and we prove that the operator is a contracting map w.r.t. the norm of

$$L^{\infty}(0,T;L^2(\Omega)) \cap L^2(0,T;H^2(\Omega))$$

2. Global estimates on the whole time interval (under further assumptions on the data): to get global estimates, independent of time, we need further regularity of the solutions

3. Extending the local solution to the whole time interval

The fixed point argument

1-
$$\widehat{s} \in \mathcal{X}_{R,T} \to \widehat{c} : \|\widehat{c}\|_{H^1(0,T;H^2(\Omega))} \leq C(R)$$

 $\partial_t c = -Ac\widehat{s} - Bc^2\widehat{s}, \text{ a.e. in } \Omega.$
 $\widehat{c}(x,t) = \frac{Ac_0(x)}{(A+Bc_0(x))e^{A\int_0^t \widehat{s}(x,\tau)d\tau} - Bc_0(x)}, \quad \text{in } \Omega \times (0,T)$
 \downarrow
2- $(\widehat{s},\widehat{c}) \to \widehat{r} : \|\widehat{r}\|_{H^1(0,T;L^2(\Gamma)} \leq C(R)$

 $\partial_t r + \xi + \Psi'(r) + G(r, \widehat{c}, \widehat{s}) = F, \, \xi \in \partial W(r)$ a.e. in (0, T)

The fixed point argument

3-
$$(\hat{c}, \hat{r}) \to s$$

 $\|s\|_{W^{1,\infty}(0,T;L^2(\Omega)) \cap H^1(0,T;H^1(\Omega)) \cap L^\infty(0,T;H^2(\Omega))} \le C(R),$
 $s \ge 0$

 $\begin{aligned} \mathbf{4-} & \|s\|_{L^{\infty}(0,T;L^{2}(\Omega))\cap L^{2}(0,T;H^{2}(\Omega))} \\ & \leq T\|s_{t}\|_{L^{\infty}(0,T;L^{2}(\Omega))} + \|s_{0}\|_{L^{2}(\Omega)} + T^{1/2}\|s\|_{L^{\infty}(0,T;H^{2}(\Omega))} \\ & \leq (T+T^{1/2})C(R) + \|s_{0}\|_{L^{2}(\Omega)} \leq R \quad \text{for small } T \end{aligned}$

The fixed point argument - the contraction

$$\begin{split} \|(s_1 - s_2)(t)\|_{L^2(\Omega)}^2 + \int_0^t \|s_1 - s_2\|_{H^1(\Omega)}^2 \\ &\leq C(R) \left(\int_0^t \|\widehat{s}_1 - \widehat{s}_2\|_{L^2(\Omega)}^2 + \int_0^t \|\widehat{s}_1 - \widehat{s}_2\|_{L^2(0,s;H^1(\Omega))}^2\right). \end{split}$$

For large j the operator \mathcal{S}^j , with $s = \mathcal{S}(\widehat{s})$ is a contraction



Some computations piecewise linear law for permeability coefficient in r



Some computations piecewise linear law for permeability coefficient in r



Some computations parabolic law for permeability coefficient in r



Some computations - r random evolution



Next steps

- Properties of the solutions and dependence on physical parameters
- Calibration and validation of the model with experimental data

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. . . .

Including more complex phenomena: combining mechanical effects and volume damage

B., Cavaterra, Freddi, Grasselli, Natalini, A nonlinear model for marble sulphating including surface rugosity: theoretical and numerical results, 2017

Biological and bio-inspired locomotion at small scales

Antonio DeSimone

SISSA-International School for Advanced Studies



http://people.sissa.it/~desimone/

Based on joint work with:

F. Alouges (Ecole Polytechnique), M. Arroyo (UPC, Barcelona), A. Beran (OGS, Trieste)

and, from SISSA:

G. Cicconofri, N. Giuliani, L. Heltai, A. Lucantonio, A. Montino, G. Noselli, M. Rossi

[G. Dal Maso, P. Gidoni, M. Morandotti]

Giornata Federazione Italiana Matematica Applicata, CNR-Roma, 27.10.17



BIOLOGICAL MACHINES Bio-inspired (swimming) micro-motility







R. Dreyfus et al.: Nature (2005)



A sperm-bot ???

Technology: how do we make it autonomous? (power source) Science: how can we steer it precisely? (direction of motion)

Antonio DeSimone, MicroMotility



Outline



Swimming -Locomotion = shape change + interaction with surrounding-medium fluid

- 1. How to generate controlled motion by prescribing time-periodic evolving shapes? (locomotion principles)
- 2. How do unicellular organisms swim?

Case study: Euglena gracilis

- 3. Metaboly in E. gracilis : how is shape controlled?
- 4. Flagellar propulsion in E. Gracilis
- Outlook





Antonio DeSimone, MicroMotility



Two well studied principles (looping in the space of shapes)





(Linear motility of human sperm cells, UWa)



(Chlamydomonas, J. Guasto)

Flagellar beating (travelling waves of shape) Sin(kx-ωt)=Cos(ωt)Sin(kx) - Sin(ωt)Cos(kx) Breast-stroke (different shapes in power/recovery parts of a stroke)

Antonio DeSimone, MicroMotility


Euglena: unicellular organism with both flagellar motility and "metaboly"





| Cell movement | Cell structure needed | Molecular motor | Motor category |
|---|---|--|-------------------------------------|
| Movements through liquid Bacterial swimming Eukaryotic swimming Metaboly | Flagella (bacterial) Cilia, flagella (eukaryotic) Unknown | Flagellar rotor (MotA/MotB) Dynein Unknown | Rotary Linear stepper Unknown |

D. Fletcher, J. Theriot: An introduction to cell motility for the physical scientists. Phys Biol. (2004)













Euglena: unicellular organism with both flagellar motility and "metaboly"





4. What about flagellar propulsion?



2D or 3D? Reconstruct 3D trajectories and shapes from 2D microscopy?



Orbits of self-propelled flagellates: a universal law







M. Rossi, **G. Cicconofri,** G. Noselli, ADS, Kinematics of flagellar swimming in Euglena gracilis: helical trajectories and flagellar shapes, PNAS, accepted.





 Euglena: can we mimic their behaviour in artificial, bio-inspired devices ? (Bio-inspired Engineering,...)

> (Soft Robotics, Bio-inspired microrobotics, Robo-Physics,... Surfaces with programmable shapes,... New smart materials and structures,....)

 Euglena: how do they really behave, and what controls their behaviour ? (Biology,...)

(Microscopic mechanisms for shape control, Gait switching,...)





"Mathematics is Biology's next microscope, only better; Biology is Mathematics' next Physics, only better." (J.A. Cohen, PLoS Biology, 2004)

Mathematical Euglena

- Mathematical microscopes: lifting of 2D microscopy data to 3D trajectories and flagellar shapes through Helix Thm
- Shape control in Biology and Bioengineering through Gauss' Thm Egregium
- Overarching conceptual paradigms: use of passive elastic properties to "explain" exquisite shape control in unicellular organisms (morphological computing, or control through constraints)



ERC Advanced Grant MicroMotility

Thank you for your attention!

Optimization and Machine Learning

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> FIMA CNR - Roma, October 27, 2017





Giornata FIMA

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Optimization for Machine Learning

- The learning problem
- The learning paradigm
- The target problem
- 2 Decomposition methods
 - Choice of the working set
 - Caching tecniques
 - Parallel
 - Numerical results





Why Machine Learning and Optimization are related ?

Optimization for Machine Learning

• Training a learning machine consists in solving an optimization problem

minimize Prediction error

- standard optimization package may directly apply to ML but exploiting both the nature and the structure of the problem must be considered
- indeed optimization algorithm, when applied to ML, must take into account other aspects rather than only convergence/speed of convergence issues

.

What is a learning problem ?

Consider a supervised classification problem.

- Given observed input-output pairs (x^p, y^p) with $x^p \in \mathbf{R}^n$, $p = 1, \dots, P$ (training set)
- Predict unknown output on test data
- It consists in finding a model $f \in \mathcal{F}$ parametrized in $\omega \in \mathbb{R}^d$ such that the prediction error is minimize

A model can be a decision tree, a neural network, a support vector machine, or others

The Expected Error

- For any new sample x, the model gives the output f(ω; x) and a measure of the error between the true unknown output y and f(x; ω) is needed
- A loss measure ℓ is used.
- The Risk $R(f, \omega)$ is the Expected Error

$$\mathbb{E}\{E(x,y;\omega)\}=\int \ell(y,f(x;\omega))d\mathcal{P}(x,y)$$

• Minimization of the Risk

$$\min_{f,\omega}$$
 Expected Error

• The optimization problem is not well-posed in the sense that the objective function is imponderable



The Empirical Error

Usually a surrogate function is used which measure the error on the known data (training set).

The Empirical Risk is the average loss

$$R_{emp}(f,\omega) = \frac{1}{P} \sum_{i=1}^{P} \ell(y^p - f(x^p;\omega))$$

The surrogate optimization problem is considered

minimize_{f,ω} Empirical Risk

Given the loss ℓ and the class of functions \mathcal{F} the *Empirical risk* depends only on the available samples In this case the empirical risk minimization problem reduces to find the optimal parameters

$$\omega^*_{emp} = \arg\min_{\omega \in \mathbb{R}^q} R_{emp}(\omega) = \sum_{p=1}^P E_i(\omega)$$

where f_{emp} is chosen.

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Tradeoff of ERM¹

Consider the error

$$\underbrace{R(f^*, \omega^*) - R(f_{emp}, \omega^*_{emp})}_{\mathcal{E}_{app} + \mathcal{E}_{est} + \mathcal{E}_{opt}}$$

- Approximation error: f* is unlikely to belong to the family F fron which f_{emp} has been selected; it measures the error due to the choice of the class F;
- Estimation error: it measures the effect of minimizing the $R_{emp}(\omega)$ instead of the expected risk $R(\omega)$ over \mathcal{F} .
- Optimization error: it reflects the fact that, when computational time is restricted, algorithms return an approximate solution $\hat{\omega}$ of min R_{emp} rather than the optimal one [1].



¹[Vapnik-Chervonenkis [8], Léon Bottou & Olivier Bousquet, 2008 [1]] 📃 🗉

Large-scale supervised machine learning: large P, large q, large n

- P: number of observations
- *q*: number of parameters
- *n*: dimension of each observation (input)

For small-scale pb, the computing time is not limited and the optimization error can be reduced to insignificant levels.

$$\underbrace{R(f^*, \omega^*) - R(f_{emp}, \omega^*_{emp})}_{\mathcal{E}_{app} + \mathcal{E}_{est}}$$

For large-scale pb, it is important to achieve a given accuracy faster otherwise the term \mathcal{E}_{opt} cannot be neglected.

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Regularized Empirical Risk

The Vapnik-Chervonenkis relationship [8] states that in probability

$$\sup_{f\in\mathcal{F}}|R(\omega)-R_{emp}(\omega)|$$

is bounded above by a function of the complexity of the class \mathcal{F} . Minimizing the empirical risk may be misleading. A regularization term is added to R_{emp} in order to "penalize complexity".

Minimization of the Regularized Empirical Risk

$$\min_{\omega \in \mathbb{R}^{q}} \underbrace{\frac{1}{P} \sum_{p=1}^{P} \ell(f(x^{p}; \omega) - y^{p})}_{\text{loss function}} + \underbrace{\mathcal{R}(\lambda, \omega)}_{\text{regularization term}}$$

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The learning paradigm

In order to specify the learning problem:

- choose the class of functions f
- choose the measure of "closeness": loss function ℓ
- choose the regularization term

The regularization term may be not enough to control the risk. Keep aside a portion of data (test set) to check actual performance.

Find ω^* by optimizing the regularized $R_{emp}(\omega)$ BUT use $R_{test}(\omega^*)$ for checking "goodness" of the solution (model f, parameters ω).

The learning problem

- 1 Cost functions are averages
- 2 No need to optimize below statistical error
- **3** Testing error is more important than training error
- 4 Avoid underfitting: look for small training error
- 6 Avoid overfitting: look for small testing error



Linear Support Vector Machine

We focus on SVM

- decision function $f = sign(w^T x + b)$
- hinge loss function

$$\ell = \max\{0, 1 - y^p(w^T x^p + b)\}$$

• mean square ℓ_2 regularized error

Linear SVM $\min_{w,b} \frac{1}{P} \sum_{p=1}^{P} \max\{0, 1 - y^p \cdot (w^T x^p + b)\} + \lambda \|w\|^2$

The regularization parameter λ must be chosen Note that a bias term *b* is present. If *n* (number of features) is small, *b* may be important. Usually *b* is not used in large-scale linear classification (internet, text).



Linear SVM

 $C \setminus I \setminus I$

The target problem can be easily stated as a convex QP in q = n + P + 1 variables.

Soft SVM

$$\min_{w,b,\xi} \quad \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{P} \xi_i$$

$$y^p(w^T x^p + b) \ge 1 - \xi_p \quad p = 1, \dots, P$$

$$\xi \ge 0$$

- It is the problem of maximizing the margin in the family of affine function with a empirical error (misclassified points $\xi > 1$) which is weighted by the hyper-parameter C
- Convex QP: Duality holds and the dual problem is usually considered for the solution.
- By duality, Kernel trick applies and nonlinear SVM appears naturally



Nonlinear SVM

For better results in classification Nonlinear SVM can be used. In this case classification is done by means of nonlinear separating surface.

The data are mapped nonlinearly in a larger space via $\Phi: \mathbb{R}^n \to \mathbb{R}^l$, a nonlinearly separating surface in \mathbb{R}^p is easily computed by a linear separator in \mathbb{R}^l .



By the kernel trick, the transformation $\boldsymbol{\Phi}$ is never required.



SVM dual Problem

In practice the quadratic convex dual problem is solved which is written as:

minimize
$$f(\alpha) = \frac{1}{2}\alpha^T Q \alpha - \mathbf{1}^T \alpha$$

subj. to $y^T \alpha = 0$,
 $0 \le \alpha \le \mathbf{1} \cdot C$

- Q ≥ 0 with elements q_{ij} = yⁱy^jk(xⁱ, x^j) where k(x_i, x_j) is the Kernel function (a scalar product in the transformed space)
- 1 ∈ Rⁿ vector of all 1's, C ∈ R user selected.
 Choice of "good" C may lead to difficulties.
 Large C may lead to overfitting (bad value of the test error).

SVM dual problem

- It is a fully dense *P* × *P* quadratic convex programming problem
- The Hessian matrix Q cannot be stored when the dimension of training set *P* is huge
- Basic operations, such as the gradient updating or the objective function evaluation, are both too time and memory consuming
- Traditional optimization methods cannot be directly employed.
- special structure of the feasible region helps in defining efficient algorithms

Decomposition methods are used

The solution of the original huge problem is obtained by solving a sequence of smaller dimension problems with the same structure of the original one.



Important issues in decomposition methods

From ML perspective

• No need to have very accurate decision function:

$$sgn\left(\sum_{p=1}^{P}\alpha_{p}^{*}y^{p}k(x^{p},x)+b\right)$$

Prediction may still be correct with a rough $\boldsymbol{\alpha}$

• Computational costs are mainly due to evaluation of the columns of the hessian matrix (kernel evaluations) which appears in gradient updating

$$\nabla f(\alpha^{k+1}) = \nabla f(\alpha^k) + \sum_{i \in W^k} Q_i(\alpha_i^{k+1} - \alpha_i^k)$$

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The number of support vectors (nonzeros in α*) << P (training size).
 It may happen that starting with α⁰ = 0, some instances are never used (shrinking techniques may be used)

Important issues in decomposition methods

From optimization perspective

• The size of W^k affects efficiency of the solution of subproblems.

How many ?

 Theoretical convergence properties depends from the rule for selecting the working set:

W^k cannot be chosen arbitrarily
Which ones ?

HOW MANY ?

The size of W^k

- $|W^k| = 2$
- Sequential Minimal Optimization (SMO)
- analytic solution α_W^*
- fast inner iteration
- larger number of outer iterations

- $|W^k| > 2$
- Usually $|W^k| = 10$
- iterative solver required
- slower inner iteration
- smaller number of outer iterations



Selection rule

To obtain convergence properties, indexes cannot be inserted in W^k as we want, BUT

MUST satisfy some rules.

Most convergent methods select indexes in the working set according to a rule tied to the violation of the Karush-Kuhn-Tucker (KKT) conditions.

When $|W^k| = 2$ this choice corresponds to the Maximal Violation Pair rule

Other rules may include second order information in the local model [current version of LIBSVM [2]] or use cyclic ordering of the indexes [4].

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Caching tecniques

In huge problems when the whole kernel matrix cannot be stored, a fraction of the memory (cache) is used to

store the most recently used hessian columns

If the indexes selected to enter the working set are already in the cache, the corresponding hessian columns need not to be re-calculated

Dynamic update of elements in the cache

when the cache is full, older elements are removed

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Caching and selection rule

To reduce the computational time it is convenient to select W^k containing elements corresponding to columns store in the cache in order to

saved Kernel computations.

However the rule that guarantees convergence properties (MVP or the more general rule)

may not allow to exploit information in the cache

How to combine SMO-type and caching strategy ?



Convergence issues

Most of the proofs of convergence of SMV algorithm relies on the analysis of sequence "compared" to the sequence generated by the MVP rule. Indeed α_{MPV}^k , solution of subproblem \mathcal{P}^k when W^k includes the two MV indeces, and the corresponding objective function value $f_{MPV}^k = f(\alpha_{MPV}^k)$ may play a significant role. As an example in [Lucidi, P.; Risi, Sciandrone 2009 [5]] an hybrid algorithm is proposed in which f_{MPV}^k is used as a reference value at each iteration. Providing that α_{MVP}^k is not at the lower or upper bounds, any point satisfying

$$f(\alpha^{k+1}) \le f_{MPV}^k - \tau \|\alpha^{k+1} - \alpha_{MVP}^k\|^p$$

can be accepted.

large degree of freedom in the choice of $lpha^{k+1}$
Hybrid Decomposition scheme



Hybrid Decomposition scheme



Hybrid Decomposition scheme



MVP, Caching and distributed architecture

- Parallel and distributed architecture [Jacobi type in [3]]
- Faster decrease of accuracy
- Exploiting SMO-type features

In [Manno, P., Sagratella [6]] a different framework for parallelize decomposition methods

Decomposition scheme (parallel)



Convergence to an optimal solution



Other technical aspect: use of a proximal point modification when needed (au > 0)



Descent iteration

Descent block

Given a feasible point α^k and $\epsilon > 0$, we say that block of variables $W_i \subseteq \{1, \ldots, n\}$ is a *descent block* at α^k if it satisfies

$$\|\alpha_{W_i}^* - \alpha_{W_i}^k\| \ge \epsilon \|\alpha_{MVP}^k - \alpha^k\|$$

- α^k_{MVP} is a solution obtained by using the MVP selection rule
- We say that k is a *descent iteration* whenever at least one descent block W_i is selected among blocks at iteration k

(How to define a descent block?)

Simplest way is to select the most violating pair in one of the blocks (but it can be relaxed)



Convergence conditions

Set $\tau_i^k > 0$ if subproblem *i* is not strictly convex

Cond.1

- Exact LS
- a descent iteration within a fixed number of its

Cond. 2

- Diminishing stepsize
- Every iteration is descent

Cond 3

- Diminishing stepsize
- a descent iteration within a fixed number of its
- $\{t^k\}$ is monotone non increasing

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A simple implementation

SMO-type implementation

- The ε -MVP always selected (block W_1^k)
- Blocks $|W_i| = 2$ with $i = 2, \ldots, q$
- $2 \cdot q$ components updated at each iteration
- Analytic stepsize
- Fast gradient update

Blocks are selected among the ε - most violating pairs. A Cache version selects blocks $i = 2, \dots, q$ within the cache.



Test problems

| name | #features | #training data | С | kernel type | cache |
|----------|-----------|----------------|---|-------------|-------|
| a9a | 123 | 32561 | 1 | gaussian | 500 |
| gisette | 5000 | 6000 | 1 | linear | 500 |
| cod-rna | 8 | 59535 | 1 | gaussian | 500 |
| real-sim | 20958 | 72309 | 1 | linear | 500 |
| rcv1 | 47236 | 20242 | 1 | linear | 500 |
| w8a | 300 | 49749 | 1 | linear | 500 |

Table : Training problems description.

The relative error RE is considered

$$\mathsf{RE} = \frac{f^* - f}{f^*}$$

where f^* is the known optimal value

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RE versus kernel evaluations

Just two examples (others has similar behaviour) (a9a and cod-rna)



We compare increasing values of pairs q:

- q = 1 LIBSVM first order and second order (dashed line) selection rule
- q = 2, 4, 8 (increasing thickness of the line)



RE versus time (cache version)





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Why Machine Learning and Optimization are related ?

Machine Learning for Optimization

- Often the analytic expression of the functions involved in the optimization problem are not known (grey/black-box optimization)
- ML model may help for obtaining well behaved surrogate models to be used to improve precision and efficiency

Consider the Organic Rankine Cycle (ORC) system for waste heat recovery application.

[Massimiani,P., Sciubba, Tocci [7]]

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Neural networks for small scale ORC optimization²



- it is a highly non-linear optimization problem
- the analytic form of the objective function and constraints not available (black box algorithms)
- free derivative algorithms, genetic algorithms, etc.



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ORC optimization problem

- Use FNN to build a surrogate models of objective and constraints
- Use standard algorithm for the optimization
- More reliable results and reduced computational time with respect to black box optimization algorithms

The maximization of the power output of the ORC system has been performed.

Decision Variables

Working fluid mass flow rate Bottom/top pressure of the ORC cycle Super-heating rate $T_5 - T_4$ Turbine efficiency Degree of regeneration $\frac{h_6 - h_9}{h_6 - h_7}$

The mathematical model

$$\max_{\mathbf{x}\in\Omega} (P_{turbine} - P_{pump}) = P_{cycle}$$

subject to
no violation of the 2nd law of thermodynamics
no deteriorations of fluid
Technical constraints

Results

The use of a Neural Network model gives

- performance slightly better (similar) to commonly used optimization algorithms
- faster computational time of the ORC optimization process
- Allow to include other constraints (Thermo-economic optimization)

.

Conclusion

Optimization for Machine Learning

- Special properties of SVM contribute to the viability of decomposition methods
- This is true also for other ML models
- A lot to do for new architecture (deep networks, Recurrent networks, Long short-term memory ecc.)

Machine Learning for Optimization

• Use of ML may help in difficult engineering problems to get reliable models, useful for both applying algorithm and to check behaviour of complex systems



Thank you for your attention



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Giornata FIMA

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Big Data and Networks for Fraud Detection in the Insurance Sector

Michele Tumminello, Andrea Consiglio

Department of Economics, Management and Statistics University of Palermo

Summary

- The Integrated Antifraud Archive
- Bipartite Networks and statistically validated networks
- Network indicators
- Criminal specialization and network motifs
- Conclusions

Big Data: The Integrated Antifraud Archive (AIA)

- Time period: 2011-2016
- About 14 million car accidents
- About 20 million individuals and companies
- About 18 million vehicles

Tumminello M, Consiglio A, **Project**: "*Network analysis and modelling of the integrated anti-fraud database*", funded by the Istituto per la Vigilanza sulle Assicurazioni (**IVASS**), which is the National Agency that supervises the activity of all the insurance companies operating in Italy. Responsible for IVASS: **Farabullini F**

Objectives

- Uncover patterns in the data that suggest fraudulent activity.
- Identify organized groups of perpetrators.

Bipartite networks



A statistical validation of co-occurrence

Suppose there are **N** events in the investigated set. We want to statistically validate the co-occurrence of subject S_A and subject S_B in **X** events against a null hypothesis of random co-occurrence. Suppose that the number of events where $S_A(S_B)$ appears is $N_A(N_B)$, whereas the number of events where both S_A and S_B appear is **X**.



The question that characterizes the null hypothesis is: <u>what is the probability</u> <u>that number X occurs</u> <u>by chance?</u>

Tumminello M, Miccichè S, Lillo F, Piilo J, Mantegna RN (2011) Statistically Validated Networks in Bipartite Complex Systems. PLOS ONE 6(3): e17994. doi:10.1371/journal.pone.0017994 http://journals.plos.org/plosone/article?id=10.1371/journal.pone.0017994

Hypergeometric distribution and Statistically Validated Networks

p-value associated with a detection of co-occurrences \ge X: p =

$$= \sum_{i=X}^{\min(N_A,N_B)} \frac{\binom{N_A}{i} \binom{N-N_A}{N_B-i}}{\binom{N}{N_B}}$$

- Count the total number of tests: T
- Arrange *p-values* in increasing order.
- Set a link between two vertices if the associated p-value satisfies one of the following inequalities



Type I error control: false positive links

Proposition: the probability that a false positive link is set in the **Bonferroni network** is smaller than α .

Co-occurrences might be dependent

Bonferroni network

- It's the most conservative statistically validated network
- The threshold is independent of p-values
- A **co-occurence** equal to **1** is not statistically significant, provided that the number of links, E, in the co-occurrence network is larger than the number of nodes, N, in the projected set, times α

$$p - value(n_{AB} = 1, N_A, N_B, N) \ge p - value(n_{AB} = 1, 1, 1, N) = \frac{1}{N} > \frac{\alpha}{E}$$

Distinguishing between subjects and vehicles

| | Nodes | Links | Connected components (CC) | Size of Iargest CC |
|---|-----------|-----------|---------------------------------|-----------------------|
| Bonferroni network of subjects * | 1,197,055 | 1,113,389 | 407.552 | 318,876 |
| Bonferroni network of vehicles* | 209,801 | 121.253 | 99,373 | 11 |

*Subjects and vehicles recorded in the white list have been excluded from the analysis

Bonferroni network of subjects: largest communities

| Community ID | Years over- expressed | Regions over-expressed | Provinces over-expressed | |
|-----------------|--------------------------|-----------------------------------|--|--|
| 1 | 2015, 2016 | SARDEGNA, LOMBARDIA, LAZIO | VA, TV, TP, TO, SS, RM, RN, RG, PO, PT, PE, PV, PD, MI, LO, LC, LT, CO, CL, CA, BG, MB, OG, VI, VR, AG | |
| 2 | 2011, 2012 | CAMPANIA*, NA | NULL, SA, AV, NA, CE | |
| 3 | - | TOSCANA*, NA | NULL, SI, PO, PT, PI, AR, LU, FI | |
| 4 | - | PIEMONTE*, VALLE_D'AOSTA | VC, TO, AT, AO, CN, BI | |
| 5 | - | BASILICATA, PUGLIA*, NA | NULL, BA, TA, PZ, MT, FG, BR, BT | |
| 6 | - | FRIULI_VENEZIA_GIULIA, VENETO* | VE, UD, TV, RO, PN, PD, FE, VI, VR, BL | |
| 7 | - | SICILIA* | TP, PA, AG | |
| 8 | - | LAZIO* | RM, RI, LT, VT | |
| 9 | - | SICILIA*, NA | NULL, SR, RG, ME, EN, CT, CL | |
| 10 | - | EMILIA_ROMAGNA* | RN, RA, OR, MO, FC, FE, BO | |
| 11 | 2015, 2016 | LAZIO* | RM, RI, LT, FR, VT | |
| 12 | 2011 | FRIULI_VENEZIA_GIULIA, VENETO | VE, UD, TV, PN, PD, NO, GO, VI, BL | |
| 13 | - | LIGURIA, NA | NULL, SV, SP, IM, GE, AL | |
| 14 | - | LAZIO, NA | NULL, RM, LT, VT | |
| 15 | 2015 | CAMPANIA* | SA, AV, NA, CE | |
| 17 | - | EMILIA_ROMAGNA*, NA | NULL, RE, PR, MO, MN, FE, BO | |
| 23 | 2016 | LOMBARDIA | VA, PV, MI, LO, LC, CR, CO, BG, MB | |
| 25 | - | LOMBARDIA, NA | PC, MN, LO, CR, BS, BG, VR | |

Are links robust to time-space localization?

An indicator of linkrobustness to localization

T=total number of events in the dataset (**T**=13,533,500 in AIA 10/2016) **B**=bonferroni threshold in the dataset (**B**=1.356e-10 in AIA 10/2016) **M**(i,j)=Min(Q) such that p-value(n(i),n(j),n(i,j),Q)<**B**

Robustness indicator

 $R(i,j) = log_{10}(T) - log_{10}(M)$

Bonferroni network: distribution of link-robustness (R>0.1)



Node (event, subject, vehicle) indicators

Subject (or vehicle) indicators

Link robustness R can be used to construct an indicator of node relevance and/or centrality

Subject strength: $s(i) = \sum_{j=1}^{N(i)} R_{i,j}$

Subject average strength: $as(i) = \frac{\sum_{j=1}^{N(i)} R_{i,j}}{N(i)}$

(relevant, weighted, and fast)

Subject betweenness: $b(i) = \sum_{p,q} \frac{\sigma_{p,q}(i)}{\sigma_{p,q}}$, where $\sigma_{p,q}$ is the number of shortest paths between p and q and $\sigma_{p,q}(i)$ is the number of those passing through i.

(relevant, unweighted, slow)

Event indicators

For any event *e*, the list *L(e)* of subject pairs with a validated connection "enhanced" by event *e* is compiled.

Event strength:
$$s(e) = \sum_{(i,j) \in L(e)} R_{i,j}$$

(relevant, weighted and fast)

Event betweenness: relevant, but unfeasible in practice (best guess)
Mixed Event-subject indicators

Statistically Validated Bipartite Network

Construction: given the SVN of subjects (or vehicles), a bipartite network is reconstructed by

- selecting from the original bipartite network all of the *event(i)*subject(j) pairs such that *event(i)* contributed to a link in the SVN between subject(j) and (at least) another subject.
- adding afterwards all of the subjects directly involved in the selected events.

K-H core of a bipartite network

The K-H core of a bipartite network is the largest bipartite **subnetwork** such that nodes of Set A have degree at least K and nodes of set B have degree at least H.





Network indicators: Mixed event-subject indicators of centrality: the K-H core

• Event oriented event-subject indicator:

 $KH_e(e,s) = \max(K)$ such that $(e,s) \in K - H$ core

• Subject oriented event-subject indicator:

 $KH_s(e, s) = \max(H)$ such that $(e, s) \in K - H$ core

• Balanced event-subject indicator:

 $KH(e,s) = \max(\sqrt{K \cdot H})$ such that $(e,s) \in K - H$ core

K-H CORE DECOMPOSITION

of a real statistically validated bipartite subnetwork



Relevant microstructures: a network of two families (in collaboration with Procura della Repubblica di Palermo)



Motifs: the heuristics

- Criminal specialization
- Some types of crime require cooperation
- Cooperating with a criminal intent requires secrecy and trust



M Tumminello, C Edling, F Liljeros, RN Mantegna, J Sarnecki (2013) The Phenomenology of Specialization of Criminal Suspects. PLoS ONE 8(5): e64703. doi:10.1371/journal.pone.0064703

Motifs and anti-fraud

Not suspicious

Suspicious



Three-node motifs: statistically validated triangles



Proposition: if random co-occurrence of three subjects, 1,2, and 3, involved in n_1 , n_2 , and n_3 events, respectively, is assumed in a dataset including N events then

$$p(n_{12}^*, n_{13}^*, n_{23}^* | n_1, n_2, n_3, N) = \sum_{n_{12}} \frac{\binom{n_1}{n_{12}} \binom{N-n_1}{n_2-n_{12}} \binom{n_1}{n_{12}-n_{12}} \binom{n_1-n_{12}}{n_{13}^*} \binom{n_2-n_{12}}{n_{23}^*} \binom{N-n_1-n_2+n_{12}}{n_3-n_{23}^*-n_{12}-n_{12}^*}}{\binom{N}{n_2}\binom{N}{n_3}}$$

$$p-value = p(n_{12}^* + n_{13}^* + n_{23}^* \ge n_{12}^{*,0} + n_{13}^{*,0} + n_{23}^{*,0})$$
²⁵

Three-node motifs and antifraud

Network of directly involved subjects (no professionals)

- Number of triangles: 162,409
- Number of statistically validated triangles:60,523

Randomly rewired network of directly involved subjects

- Average number of triangles: 18,535
- Average Number of statistically validated triangles: 0.08

Final Remarks

- 1. The network of subjects and vehicles carry different information.
- 2. Introduced network indicators and IVASS subject indicators carry complementary information, and, therefore, can fruitfully be integrated.
- 3. The test on "claims closed following investigation" and the analysis of a few case studies indicate the effectiveness of the overall approach.
- 4. Introduced network indicators will be operative by Jan 2018.
- Next steps: (a) integrating three-node motifs in the SVN (exp. Jun 2017); (b) developing an integrated indicator (exp. end 2018);

Thanks!

Michele Tumminello

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TURNING THE MAP INTO A GLOBE LOGIC IN FORECASTING AND DECISION MAKING

Hykel Hosni



Associazione Italiana di Logica e Applicazioni



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Giornata FIMA - CNR Roma 27/10/2017

Hykel Hosni (AILA-UniMi)

Logic and uncertainty

27/10/11 1 / 23

A COMPREHENSIBLE MISUNDERSTANDING



but a misunderstanding nonetheless!

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Logic and uncertainty

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A VERY LONG STORY, SHORT

BEGRIFFSSCHRIFT,

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D* GOTTLOB FREEE, PREATORNESS DER KATHENALIK AN DER UNWERTÄT JESA.

HALLE 46. VERLAG VON LOUIS NEBERT. 1879. AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

BY

GEORGE BOOLE, LL. D. PROFESSOR OF MATHEMATICS IN QUEEN'S COLLEGE, CORK.



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27/10/11

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Hykel Hosni (AILA-UniMi)

Logic and uncertainty

A VERY LONG STORY, SHORT



1980s - Today

AI thrives on combining Fregean rigour and Boolean ambition

Hykel Hosni (AILA-UniMi)

Logic and uncertainty

AN INVESTIGATION

OF

THE LAWS OF THOUGHT,

ON WHICH ARE FOUNDED

THE MATHEMATICAL THEORIES OF LOGIC AND PROBABILITIES.

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THRU' THE LOGICIANS' GLASS ...

 $P: \mathcal{SL} \to [0,1]$ is a *probability function* on \mathcal{L} if it satisfies NORMALISATION

$$\models \theta \Rightarrow P(\theta) = 1$$

Additivity

$$\models \neg(\theta \land \phi) \Rightarrow P(\theta \lor \phi) = P(\theta) + P(\phi)$$

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THRU' THE LOGICIANS' GLASS

 $P: \mathcal{SL} \to [0,1]$ is a probability function on \mathcal{L} if it satisfies NORMALISATION

$$\models \theta \Rightarrow P(\theta) = 1$$

ADDITIVITY

$$\models \neg(\theta \land \phi) \Rightarrow P(\theta \lor \phi) = P(\theta) + P(\phi)$$

... and what is to be found there

DIAGNOSIS

 \models is liable for unwelcome features

- "uncertainty \neq ignorance"
- ► SAT is NP-complete

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Logic and uncertainty

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TREATMENT Use a better logic!

- Why is this relevant?
- **2** Logic in action
- Ourrent initiatives

Probability and its critics

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Logic and uncertainty

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THE ARGUMENT FROM INFORMATION

The sense in which I am using the term ['uncertain' knowledge] is that in which the prospect of a European war is uncertain [...] About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know.



J. M. Keynes, *The General Theory of Employment* Q.J.E. 51 (1937): 209-23.

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The sense in which I am using the term ['uncertain' knowledge] is that in which the prospect of a European war is uncertain [...] About these matters there is no scientific basis on which to form any calculable probability whatever. We simply do not know.

Nevertheless, the necessity for action and for decision compels us as practical men to do our best to overlook this awkward fact and to behave exactly as we should if we had behind us a good Benthamite calculation of a series of prospective advantages and disadvantages, each multiplied by its appropriate probability, waiting to be summed.



J. M. Keynes, *The General Theory of Employment* Q.J.E. 51 (1937): 209-23.

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THE ARGUMENT FROM TRACTABILITY

A person required to risk money on a remote digit of π would have to compute that digit in order to comply fully with the theory though this would really be wasteful if the cost of computation were more than the prize involved. For the postulates of the theory imply that you should behave in accordance with the logical implications of all that you know. Is it possible to improve the theory in this respect, making allowance within it for the cost of thinking, or would that entail paradox $[\ldots]$? If the remedy is not in changing the theory but rather in the way in which we attempt to use it, clarification is still to be desired.



L. J. Savage (1967),"Difficulties in the theory of personal probabilities"

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Logic in action

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GOOD FORECASTS SHOULD BE (AT LEAST)

- ► Coherent
- ► Accurate
- Useful

Suppose

- θ : "Italy's GDP for 2017 will increase by 2.5%"
- ϕ : "Average pay for fresh gradates in 2020 will be >1000 E"

To evaluate the *coherence* and *accuracy* of forecasts such as $P(\theta \land \phi)$, $P(\neg \phi)$, etc. we need to be able to resolve the relevant uncertainty (i.e. wait until we can assign a truth-value to the relevant sentences/events) according to well defined logical rules.

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THE CLASSICAL CASE

Let $\theta_1, \ldots, \theta_n$ be events in a logical language \mathcal{L} and \mathbb{V} the set of CLASSICAL logical valuations $v : \mathcal{L} \to \{0, 1\}$, i.e. which OBEY THE STANDARD BOOLEAN TABLES. The assessment

$$\Phi: \theta_1 \mapsto \alpha_1, \ldots, \theta_n \mapsto \alpha_n,$$

where $\alpha_i \in [0, 1], i = 1, ..., n$. is *coherent* if real numbers $\sigma_1, ..., \sigma_n$ cannot be found such that for every valuation v

$$\sum_{i=1}^{n} \sigma_i(\alpha_i - v(\theta_i)) < 0.$$

THEOREM (DE FINETTI 1931)

An assessment Φ is **coherent** iff there is a probability distribution P which extends it, that is, $P(\theta_i) = \alpha_i$ for i = 1, ..., n.

Hykel Hosni (AILA-UniMi)

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Image: A marked by the second seco

When 2 is not enough



(1) The Spanish prime minister, Mariano Rajoy, has refused to rule out imposing direct rule over Catalonia. Photograph: Angel Diaz/EPA

▶ The many banks and firms which moved out of Barcelona after the referendum made presumably coherent forecasts nonetheless!

Hykel Hosni (AILA-UniMi)

Logic and uncertainty

27/10/11 13 / 23

3

Partially-resolving uncertainty is typical when considering *informational events*.

$$v(\theta) = \begin{cases} 1 \text{ if I am informed that } \theta \\ 0 \text{ if I am informed that } \neg \theta \\ \bot \text{ if I am not informed about } \theta \end{cases}$$

Recently a family of logics based on this semantics have been developed, which *as a bonus* provide tractable approximations of \models **more**

D' Agostino, M., Finger, M., & Gabbay, D. (2013). Semantics and proof-theory of depth bounded Boolean logics. *Theoretical Computer Science*, 480, 43-68.

Hykel Hosni (AILA-UniMi)

IPCC 2007 Summary for Policymakers

"By 2080, an increase of 5 to 8% of arid and semi-arid land in Africa is projected under a range of climate scenarios (high confidence)."

Multiple layers of uncertainty:

- "arid" and "semi-arid" are clearly *vague* concepts
- ▶ the expected increase is given in the form of an interval
- ▶ the uncertainty of the overall statement is quantified with "high confidence"

 \models is clearly inadequate to model the uncertainty resolution mechanism in cases like this

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LUKASIEWICZ LOGIC (AND MV-ALGEBRAS)

An illustration of the setup

- ▶ Truth values range over [0, 1]
- ► For each connective there exists a fixed function which determines the truth-value of complex sentences:

$$F_{\neg}(x) = 1 - x$$

$$F_{\wedge}(x, y) = \max\{0, x + y - 1\}$$

$$F_{\vee}(x, y) = \min\{1, x + y\}$$

$$F_{\rightarrow}(x, y) = \min\{1, 1 - x + y\}$$

▶ normalised and additive measures (called *states*) are defined on the resulting logic

D. Mundici, Advanced Lukasiewicz calculus and MV-algebras, Springer 2011.

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Logic and uncertainty

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Current initiatives

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RESEARCH

Lorentz center

Models of Bounded Reasoning in Individuals and Groups, Lorentz Center, Leiden. 2-6 July 2018, co-organised with

- ▶ D. Grossi, Computer Science, Liverpool
- ▶ S. Modgil, Computer Science, KCL



Summer School on Logic, Uncertainty and Games, Lake Como School of Advanced Studies, 9-13 July 2018, co-organised with

- ▶ P. Battigalli, Decision Sciences, Bocconi
- ▶ M. Marinacci, Decision Sciences, Bocconi

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TEACHING

Programs > Undergraduate School > Prospective Students > Economic and Social Sciences > Program Structure

- Program Structure
 - » Educational Objectives
 - » Study Plan
 - » Intended Learning Outcomes
 - » Career Opportunities

Program structure - Economic and Social Sciences

FIRST YEAR

Main topics

- > Economics (micro and macro)
- > Math (theoretical and applied)
- > Logic and Methods for Social Sciences
- > Management

First foreign language Computer science

During the first year, you will be given training in several basic subjects in order to understand how companies and organizations work. (Management), as well as the economic system in which they operate. Economics, in fact, describes the functioning of economic systems both at an overall level (macroeconomics) and at the level of the individual behavior of firms and consumers (microeconomics). Through Math and Statistics, both theoretical and applied, you will be provided with a wide set of quantitative tools to understand and apply an array of economic models. Finally, the course in Logic and Methodology of Social Sciences gives you the fundamentals of logic and critical thinking to understand how the mind should work in order to detect failacious patterns of reasoning (when dons B follow from A?), assess and (de)construct arguments, think consistently and analytically.



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Logic and uncertainty

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References

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- Video Podcasts on https://youtu.be/fKyETgxdbM8
- ▶ Volume 2 out in March 2018

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Logic and uncertainty

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27/10/11

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- M. D'Agostino, T. Flaminio, and H. Hosni. "Rational beliefs real agents can have A logical point of view". In: Journal of Machine Learning Research: Workshops and Conference and Proceedings 58 (2017), pp. 1–13.
- T. Flaminio, H. Hosni, and S. Lapenta. "Convex MV-Algebras: Many-Valued Logics Meet Decision Theory". In: *Studia Logica* Online First (2017).
- T. Flaminio, H. Hosni, and F. Montagna. "Strict coherence on many valued algebras". In: *Journal of Symbolic Logic* (in print) (2017).

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INFORMATIONAL SEMANTICS



The hierarchy of tractable approximations of \models

- $\blacktriangleright \models_0$ is defined by taking the closure under the informational tables
- ▶ $\models_k, i \in \mathbb{N}$ is built in discrete steps by *assuming* that a sentence which was left undecided at level k 1 is now taking either 0 or 1.
- Each k yields a polynomial decision procedure
- $\triangleright \models_{\infty}$ coincides with \models



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